8. A patient is tested for *aplastic anemia*, a medical condition that afflicts 1% of the population. The test is 95% accurate. Suppose the test result for the patient is positive, i.e., the test claims that the patient does have the disease. What is the probability that the patient is indeed affected? [**Hint:** Define the two events $D = \{\text{Patient is affected}\}$ and $T = \{\text{Test result is positive}\}$. Here *accuracy* means that both $P(T|D) = 0.95$ and $P(\overline{T}|\overline{D}) = 0.95$. The quantity $P(T|D)$ is known as the *sensitivity* or true positive rate of the test, and $P(\overline{T}|\overline{D})$ is known as the *specificity* or true negative rate. Observe that D and $\overline{D}$ are mutually exclusive and exhaustive, and we want $P(D|T)$.] [**Answer:** $\frac{0.95\times 0.01}{0.95\times 0.01+0.05\times 0.99} \approx 0.16$.]

9. Two urns, urn 1 and urn 2, contain, respectively, two white and one black balls, and one white and five black balls. One ball is transferred from urn 1 to urn 2 and then one ball is drawn from the latter. Suppose the drawn ball happens to be white. What is the probability that the transferred ball was black? [**Hint:** Define the two events $A = \{\text{Drawn ball is white}\}$ and $B = \{\text{Transferred ball is black}\}$ and note that $P(A|B) = 1/7$ (why?), $P(A|\overline{B}) = 2/7$ (why?). We need $P(B|A)$.] [**Answer:** $\frac{\frac{1}{7}\cdot\frac{1}{3}}{\frac{1}{7}\cdot\frac{1}{3}+\frac{2}{7}\cdot\frac{2}{3}} = \frac{1}{5}$.]

10. A box contains three coins with a head on each side, four coins with a tail on each side, and two fair coins. One of these nine coins is selected at random and tossed once.

    (a) What is the probability that a head will be obtained? [**Answer:** $1\times\frac{3}{9}+0\times\frac{4}{9}+\frac{1}{2}\times\frac{2}{9} = \frac{4}{9}$.]

    (b) Suppose that the toss does result in a head. What is the probability that the coin used for tossing was fair (i.e., not with a head on each side)? [**Answer:** $\frac{1}{4}$.]

11. (WMS, Problem 2.134) Two methods, A and B, are available for teaching a certain industrial skill. The failure rate is 20% for A and 10% for B. However, B is more expensive and hence is used only 30% of the time. (A is used the other 70%.) A worker was taught the skill by one of the methods but failed to learn it correctly. What is the probability that she was taught by method A? [**Answer:** $\frac{14}{17}$.]

12. (WMS, Problem 2.132) Using the law of total probability or otherwise, prove the following:

    (a) If $P(A|B) = P(A|\overline{B})$, then A and B are independent. [**Hint:** $P(A) = P(A|B)P(B) + P(A|\overline{B})P(\overline{B})$].

    (b) If $P(A|C) > P(B|C)$ and $P(A|\overline{C}) > P(B|\overline{C})$, then $P(A) > P(B)$.

13. (WMS, Problem 3.7.) Each of three balls are randomly placed into one of three bowls. Find the PMF and DF of $X =$ the number of empty bowls. [**Hint:** Observe that here $\mathscr{X} = \{0, 1, 2\}$. Now, $p_X(1) = P(X = 1)$ is a bit tricky to calculate, but recall that $\sum_{x\in\mathscr{X}} p_X(x) = 1$.]. [**Answer:** $p_X(0) = \frac{6}{27}, p_X(2) = \frac{3}{27}$, and $p_X(1) = 1 - p_X(0) - p_X(1) = \frac{18}{27}$.]

14. Suppose that a random variable X has a discrete distribution with the following PMF:

$$p_X(x) = \begin{cases} cx & \text{for } x = 0, \dots, 5 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the value of the constant c and calculate $E(X)$. [Answer: $c = \frac{1}{15}, E(X) = \frac{11}{3}$.]

15. (WMS, Problem 3.10.) A single cell can either die, with probability 0.1, or split into two cells, with probability 0.9, producing a new generation of cells. Each cell in the new generation dies or splits into two cells independently with the same probabilities as the initial cell. Find the probability distribution for the number of cells in the next generation. (Note that the number of cells cannot be odd.) What is the expected number of cells in the next generation? [**Answer:** $p(0) = 0.1 + 0.9 \times 0.1 \times 0.1 = 0.109$, $p(4) = 0.9 \times 0.9 \times 0.9 = 0.729$ and $p(2) = 1 - p(0) - p(4) = 0.162$.]