1. (WMS, Problem 3.118.) Five cards are dealt at random and without replacement from a standard deck of 52 cards. What is the probability that the hand contains all 4 aces if it is known that it contains at least 3 aces? [**Answer:** $\frac{\binom{4}{4}\binom{48}{1}}{\binom{4}{4}\binom{48}{1}+\binom{4}{3}\binom{48}{2}}$.]

2. (WMS, Problem 3.135.) A salesperson has found that the probability of a sale on a single contact is approximately 0.03. If the salesperson contacts 100 prospects, what is the probability of making at least one sale? [**Answer** $\approx 1 - e^{-3}$.]

3. An airline sells 200 tickets for a certain flight on an airplane that has only 198 seats because, on the average, 1 percent of purchasers of airline tickets do not appear for the departure of their flight. Determine the probability that everyone who appears for the departure of this flight will have a seat. [**Hint:** Define X = # people who do not appear for their flight.] [**Answer** $\approx 1 - 3e^{-2}$.]

4. Suppose a discrete RV X with support $\mathscr{X} = \{-N, -(N-1), \cdots, -1, 0, 1, \cdots, N-1, N\}$ is symmetric (about 0), i.e., $P(X = k) = P(X = -k)$ for all k.
   (a) Show for any odd positive integer r, $\mu'_r = 0$, where μ'_r denotes the r-th raw moment of X.
   (b) Prove that for any k, $\mu_k = \mu'_k$, where μ_k denotes the k-th central moment of X. Thus, μ_r is also zero when r is an odd positive integer.

5. (WMS, Problem 3.147 - 3.148.) Let Y have a geometric distribution with probability of success p, and define $q = 1 - p$.
   (a) Show that the MGF for Y is $M_Y(t) = \frac{pe^t}{1-qe^t}$.
   (b) Differentiate the MGF in part (a) to find $E(Y)$ and $E(Y^2)$. Then find $V(Y)$.

6. First, a result: If X is a nonnegative RV with finite expectation and $a > 0$, then

$$P(X \geq a) \leq \frac{E(X)}{a} \quad \text{(Markov inequality).}$$

   Using the above inequality, prove Chebyshev's theorem: if Y is a RV with mean μ and finite variance σ^2, then, for any constant $k > 0$,

$$P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}.$$

7. This exercise demonstrates the *tightness* of Chebyshev's theorem. For any constant k, define a RV X with support $\mathscr{X} = \{-1, 0, 1\}$ and PMF $p_X(-1) = p_x(1) = \frac{1}{2k^2}$ and $p_X(0) = 1 - \frac{1}{k^2}$.
   (a) Verify that p_X is indeed a PMF.
   (b) Find the mean μ_X and variance σ_X^2 of X.
   (c) Show that $P(|X - \mu_X| \geq k\sigma_X) = \frac{1}{k^2}$. [**Note:** Thus, for any positive constant k, we can construct a RV X (and hence a probability distribution), for which equality holds in the Chebyshev theorem.]

8. A RV X with support $\mathscr{X} = \{1, 2, \cdots, N\}$ is said to follow a discrete uniform distribution (over the set of first N positive integers) if all elements of the support $\mathscr{X}$ are equally probable as values of X, i.e., if $P(X = x) = c$, for some constant c and for all $x \in \mathscr{X}$.

(a) Find c. [Answer: $c = \frac{1}{N}$.]

(b) Find $E(X)$ and $V(X)$. The following formulas should be helpful:

$$1 + 2 + \cdots + N = \frac{N(N+1)}{2}$$
$$1^2 + 2^2 + \cdots + N^2 = \frac{N(N+1)(2N+1)}{6}$$

[**Answer:** $E(X) = \frac{N+1}{2}, V(X) = \frac{N^2-1}{12}$.]

9. (WMS, Problem 4.8.) Suppose that Y has PDF

$$f(y) = \begin{cases} ky(1-y), & 0 \leq y \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the value of k that makes $f(y)$ a probability density function. [**Answer:** $k = 6$.]

(b) Find the CDF $F(y)$ of Y. [**Partial answer:** $F(y) = y^2(3 - 2y)$ for $0 \leq y < 1$.]

(c) Calculate $P(0.4 \leq Y < 1)$.

(d) Calculate $P(Y \leq 0.4 \mid Y \leq 0.8)$ and hence find $P(Y \geq 0.4 \mid Y \leq 0.8)$.

10. (WMS, Problem 4.19.) Let the DF of a random variable Y be

$$F(y) = \begin{cases} 0, & y \leq 0 \\ \frac{y}{8}, & 0 < y < 2 \\ \frac{y^2}{16}, & 2 \leq y < 4 \\ 1, & y \geq 4. \end{cases}$$

(a) Find the PDF of Y. [**Partial answer:** $f(y) = \frac{1}{8}$ for $0 < y < 2$ and $f(y) = \frac{y}{8}$ for $2 \leq y < 4$.]

(b) Find $P(1 \leq Y \leq 3)$.

(c) Find $P(Y \geq 1.5)$.

(d) Find $P(Y \geq 1 | Y \leq 3)$.