1. Suppose X is a RV with $E(X) = 2$ and $V(X) = 4$. Find $E(X + 2)^2$. [**Answer:** 20.]

2. (WMS, Problem 4.28.) The proportion of time per day that all checkout counters in a supermarket are busy is a RV Y with PDF

$$f(y) = \begin{cases} cy^2(1-y)^4, & 0 \le y \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Find the value of c that makes $f(y)$ a probability density function. [**Answer:** $c = 105$.]

   (b) Find $E(Y)$. [**Answer:** 3/8.]

3. An absolutely continuous RV X with PDF f is called symmetric (about 0), if for every $x \in \mathbb{R}$, $f(x) = f(-x)$.

   (a) Assuming it exists, show that $E(X) = 0$.

   (b) If F denotes the DF of X, show that $F(0) = \frac{1}{2}$. Hence find the median of X. (You can assume F to be strictly increasing on a neighborhood of 0.)

   [**Hint:** Use the properties of odd and even functions.]

4. (WMS, Problem 4.30.) The proportion of time Y that an industrial robot is in operation during a 40-hour week is a random variable with probability density function

$$f(y) = \begin{cases} 2y, & 0 \le y \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Find $E(Y)$ and $V(Y)$. [**Answer:** $E(Y) = 2/3, V(Y) = 1/18$.]

   (b) For the robot under study, the profit X for a week is given by $X = 200Y - 60$. Find $E(X)$ and $V(X)$.

   (c) Find an interval in which the profit should lie for at least 75% of the weeks that the robot is in use. [**Answer:** [20.9476, 167.6142].]

5. (WMS, Problem 4.47.) The failure of a circuit board interrupts work that utilizes a computing system until a new board is delivered. The delivery time, Y, is uniformly distributed on the interval one to five days. The cost of a board failure and interruption includes the fixed cost c_0 of a new board and a cost that increases proportionally to Y^2. If C is the cost incurred, $C = c_0 + c_1 Y^2$. (c_1 is a constant.)

   (a) Find the probability that the delivery time exceeds two days. [**Answer:** 0.75.]

   (b) In terms of c_0 and c_1, find the expected cost associated with a single failed circuit board.

6. Recall that the MGF of a RV X is defined by $M_X(t) := E(e^{tX})$. Hence, for continuous X, $M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$, f being the PDF of X. From this definition, find the MGF of X when $X \sim \text{Exp}(\beta)$. Hence (by differentiating) find $E(X)$ and $E(X^2)$. Then find $V(X)$. [**Partial Answer:** $M_X(t) = 1/(1 - \beta t)$, provided $t < 1/\beta$.]

7. (WMS, Problem 4.97 -4.98.) A manufacturing plant uses a specific bulk product. The amount of product used in one day can be modeled by an exponential distribution with $\beta = 4$ (measurements in tons).

(a) Find the probability that the plant will use more than 4 tons on a given day.

(b) How much of the bulk product should be stocked so that the plants chance of running out of the product is only 0.05?

8. (WMS, Problem 4.91.) Let Y have an exponential distribution with $P(Y > 2) = .0821$. Find $E(Y)$ and $P(Y \leq 1.7)$. [**Partial Answer:** $E(Y) \approx 0.8$.]