Solve the following problems. Be sure to show all work and prove all statements.

1. Let $\xi$ and $\eta$ be two vector bundles over a paracompact base space $X$. Explicitly prove that the space of bundle morphisms $\text{hom}(\xi, \eta)$ has the structure of a bundle over $X$.

2. (MS 4a) Prove
\[ w_k(\xi \times \eta) = \sum_{i=0}^{k} w_i(\xi) \cup w_{k-i}(\eta). \]

3. Prove that $\mathbb{R}P^2 \times \mathbb{R}P^2$ is not cobordant to $\mathbb{R}P^4$.

4. Prove that the standard torus $T$ is cobordant to $S^2$.

5. Suppose $k_1 \neq k_2$ and $k_1 + j_1 = k_2 + j_2$ for some $j_1$ and $j_2$. Prove $S^{k_1} \times S^{j_1}$ is cobordant to $S^{k_2} \times S^{j_2}$.

6. Two real vector bundles $\xi$ and $\eta$ over $X$ are stably isomorphic if there exists an $n$ such that $\xi \oplus \varepsilon^n \cong \eta \oplus \varepsilon^n$, where $\varepsilon^n = X \times \mathbb{R}^n$ is the trivial $n$-plane bundle over $X$. If $\xi$ and $\eta$ are stably isomorphic, prove $w_i(\xi) = w_i(\eta)$ for every $i$.

7. Prove that the orthogonal group $O(n + k)$ acts transitively on $G_n(\mathbb{R}^{n+k})$. Identify the stabilizer of the $n$-plane $\mathbb{R}^n \oplus 0 \subset \mathbb{R}^n \times \mathbb{R}^k$ under this action; call it $S$. Show $G_n(\mathbb{R}^{n+k}) \cong O(n + k)/S$. 
