

# MTG 7396: Advanced Topics in Topology

## Differential Topology and Vector Bundles

MWF Period 7, 1:55 - 2:45, Little Hall 223

Fall 2017

**Instructor:** Michael Catanzaro   **Email:** catanzaro@ufl.edu   **Office Location:** 411 Little Hall  
**Office Hours:** M Period 5, 11:45 - 12:35, W Period 4, 10:40 - 11:30, F Period 6 12:50 - 1:40

**Prerequisites** MTG 6347, or a basic understanding of algebraic topology and permission of the instructor.

**Description and Goals.** In this course, we will explore a variety of concepts related to differential topology and vector bundles. We will study vector bundles for their own sake, as well as their usefulness outside of algebraic topology. This will include a brief exploration of analysis on manifolds and de Rham theory. We will then classify vector bundles, and their associated principal bundles, using the bundle classification theorem. Our ultimate goal is to describe bundles using the language of characteristic classes with plenty of examples.

**Course Outline.** We will begin with the basics of differential topology, including smooth manifolds, tangent spaces, tangent bundles, and their duals. Using the tangent bundle as our guide, we will define general vector bundles and discuss their topological properties and classification. We will then define characteristic classes, and use them to say as much as we can about bundles. In particular, we will study Stiefel-Whitney, Chern, and the Euler class in detail. Accompanying this will be the theory of orientations of bundles, focusing on the Thom class. We will develop the theory and language of topological K-theory along the way. Time permitting, we will discuss principal G-bundles, Swan's theorem relating topological and algebraic K-theory, Chern-Weil theory, Gerbes, and the Atiyah-Singer index theorem.

**Resources.** There is no required text for this course. All material needed will be explicitly given in class unless otherwise noted. However, there is a multitude of texts written on the topics we will cover, and additional reading may be helpful. The following resources may be useful to read:

- **Characteristic Classes**, by Milnor and Stasheff,
- **Introduction to Differential Topology**, by Brocker and Janich,
- **Differential Topology**, by Guillemin and Pollack, and
- **From Calculus to Cohomology**, by Madsen and Tornehave.

**Evaluation and Grading.** There will be several homework assignments throughout the course. Some of these problems will be very difficult and will require a significant amount of time to work.

I encourage you to collaborate with your classmates and ask questions in class. In addition, there will be a take-home final exam.

Grades will be computed with homework accounting for **75%** of your grade, and the take-home final worth **25%** of your grade. The following grading scale is employed.

Grading Scale out of 100%								
100-97	A	89-87	B+	79-77	C+	69-67	D+	
96-93	A	86-83	B	76-73	C	66-63	D	59-0 F
92-90	A-	82-80	B-	72-70	C-	62-60	D-	

**Attendance Policy:** I expect you to come to class each day; this is critical for success in a mathematics course. If you must miss class or an exam for a religious observance, you must notify me in writing of participation in this observance at least one week prior to the event. This is consistent with the university policies that can be found here: <https://catalog.ufl.edu/ugrad/current/regulations/info/attendance.aspx>.

**Course Policy:** The use of calculators on any in-class exam is prohibited in this course. Further, written medical documentation is required for any make-up exam.

**Academic Integrity:** UF students are bound by The Honor Pledge which states, “We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honor and integrity by abiding by the Honor Code”. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: “On my honor, I have neither given nor received unauthorized aid in doing this assignment.” The Honor Code, which can be found here <http://www.dso.ufl.edu/sccr/process/student-conduct-honor-code/>, specifies a number of behaviors that are in violation of this code and the possible sanctions. Furthermore, you are obligated to report any condition that facilitates academic misconduct to appropriate personnel. If you have any questions or concerns, please consult with the instructor.

**Students with Disabilities:** Students requesting classroom accommodation must first register with the Dean of Students Office. The Dean of Students Office will provide documentation to the student who must then provide this documentation to the Instructor when requesting accommodation. The Disability Resource Center can be contacted at (352)-392-8565 or by visiting <https://www.dso.edu/drc/>. Students should provide the DRC with the appropriate documentation as early as possible in the semester.