MAA 4102, MAA 5104
Homework 10
Due: Friday, March 31, 2017

Solve all problems and be sure to show all work. Answers with no supporting work will be given no credit.

1. (p.131 3.2.6) Consider the function $f(x) = \frac{x^2-1}{x-1}$.
   (a) Evaluate $\lim_{x\to 1} f(x)$, if possible. Explain clearly.
   (b) Graph the function f.

2. (p.132 3.2.8) Consider the function $f : [-1, 1] \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 0 & x = \pm\frac{1}{n} \text{ for } n \in \mathbb{N}, \\ 1 & \text{otherwise.} \end{cases}$$

   Find the given limits if possible and prove your results.
   (a) $\lim_{x\to 5/8} f(x)$
   (b) $\lim_{x\to -1/3} f(x)$
   (c) $\lim_{x\to 0} f(x)$.

3. Assuming that all of the limits involved are finite, prove or disprove the following.
   (a) $\lim_{x\to 4a} f(x) = 4 \lim_{x\to a} f(x)$
   (b) $\lim_{x\to a} f(4x) = 4 \lim_{x\to a} f(x)$
   (c) $\lim_{x\to a} f(x) = 4 \lim_{x\to a} f(\frac{1}{4}x)$
   (d) $\lim_{x\to 4a} f(x) = \lim_{x\to a} f(4x)$

4. Let $D = \{1/n \mid n \in \mathbb{N}\}$ and $f(x) = -x^2 + 1$. Evaluate $\lim_{x\to -1} f(x)$, $\lim_{x\to 1} f(x)$, and $\lim_{x\to 0} f(x)$, if possible. Explain your reasoning.