

**MAA 4102, MAA 5104**  
**Homework 12**  
**Due: Friday, April 14, 2017**

Solve all problems and be sure to show all work. Answers with no supporting work will be given no credit.

1. (p.153 4.1.3) Determine where the given functions are continuous. Explain clearly.

(c)  $f : \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = \sin(x)/x$  if  $x \neq 0$ , and  $f(0) = 1$ .

(i)  $f : (0, \infty) \rightarrow \mathbb{R}$ , given by  $f(x) = 1/\sqrt{x}$ .

(j)  $f : \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = 1$  if  $x \in \mathbb{Q}$  and  $f(x) = 0$  if  $x \in \mathbb{R} \setminus \mathbb{Q}$ .

2. (p.154 4.1.5c) Let  $f, g : D \rightarrow \mathbb{R}$  be functions continuous at  $x = a$ . Let

$$\max(f, g)(x) = \begin{cases} f(x) & \text{if } f(x) \geq g(x) \\ g(x) & \text{if } f(x) < g(x) \end{cases}$$

be the maximum function of  $f$  and  $g$ . Prove that  $\max(f, g)$  is continuous at  $x = a$ .

3. (p.154 4.1.6) Prove or find a counterexample to the following statements. Assume  $f$  and  $g$  are defined on the indicated intervals.

(a)  $f$  bounded on  $[a, b]$  implies that  $f$  is continuous on  $[a, b]$ .

(e)  $f$  and  $g$  not continuous on  $(a, b)$  implies that  $f + g$  is not continuous on  $(a, b)$ .

(h)  $fg$  and  $f$  continuous on  $(a, b)$  implies that  $g$  is continuous on  $(a, b)$ .

(l)  $f$  defined on  $[a, b]$  and  $\{f(x_n)\}$  converges to  $f(c)$  for any sequence  $\{x_n\}$  in  $[a, b]$  converging to  $c \in [a, b]$  implies that  $f$  is continuous at  $x = c$ .

4. (p.160 4.2.1) Do parts c,d,h, and i.