MAA 4102, MAA 5104
Homework 2
Due: Monday, January 20, 2017

Solve all problems and be sure to show all work. Answers with no supporting work will be given no credit.

1. Determine whether the given function is an injection, surjection, bijection, or none of these. Clearly explain your answers.
   (a) $f : (0, \infty) \to \mathbb{R}$, given by $f(x) = \frac{1}{x}$.
   (b) $g : \mathbb{R} \to (0, \infty)$, given by $g(x) = e^x$.
   (c) $h : \mathbb{R} \to \mathbb{R}$, given by $h(x) = x^2 + x + 1$.

2. Determine whether the given function is (strictly) increasing, (strictly) decreasing, bounded, bounded above, or bounded below. Also find the supremum, infimum, maximum, and minimum, if they exist.
   (a) $f : [0, \infty) \to \mathbb{R}$, given by $f(x) = 2|x| - x^2$.
   (b) $g : \mathbb{R} \to \mathbb{R}$, given by $g(x) = \sqrt[3]{x} - x$.
   (c) $h : \mathbb{N} \to \mathbb{R}$, given by $h(n) = \frac{1}{n}$.
   (d) $j : (0, \infty) \to \mathbb{R}$, given by $j(x) = \sin x \cos x$.

3. (p. 19, 1.2.13) Prove the following.
   (a) If both functions $f : A \to B$ and $g : B \to C$ are one-to-one, prove that the composition $g \circ f : A \to C$ is one-to-one.
   (b) If both functions $f : A \to B$ and $g : B \to C$ are onto, prove that the composition $g \circ f : A \to C$ is onto.

4. (p.20, 1.2.20) Suppose that $f : X \to Y$, with $A, B \subset Y$. Prove the following.
   (a) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.
   (b) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.

5. (p. 20, 1.2.21) Suppose that $f : X \to Y$ with $A \subset X$ and $B \subset Y$. Prove the following.
   (a) $A \subset f^{-1}(f(A))$.
   (b) $f(f^{-1}(B)) \subset B$.