MAA 4102, MAA 5104
Homework 3
Due: Friday, January 27, 2017

Solve all problems and be sure to show all work. Answers with no supporting work will be given no credit.

1. Use mathematical induction to prove the following statements.

   (a) (p.29 1.3.2 (d))
   $$\sum_{k=1}^{n}(2k-1) = n^2\,.$$

   (b) (p. 29, 1.3.2 (n)) If $x_1, x_2, \ldots, x_n$ are n real numbers in the interval $[a, b]$, then
   $$a \le \frac{x_1 + x_2 + \cdots + x_n}{n} \le b$$
   for all $n \in \mathbb{N}$.

   (c) (p. 29, 1.3.2 (u)) For $n \ge 2$,
   $$\sum_{k=1}^{n}\frac{1}{k^2} \le \frac{7}{4} - \frac{1}{n}\,.$$

   (d) Suppose $a_1 = 1$, $a_2 = 8$, and $a_n = a_{n-1} + 2a_{n-2}$ for $n \ge 3$. Prove that
   $$a_n = 3 \cdot 2^{n-1} + 2 \cdot (-1)^n\,.$$

2. Prove that if q^2 is divisible by 5, then q is divisible by 5, for any $q \in \mathbb{Z}$.

3. Negate the following statements.

   (a) There exists $c \in \mathbb{R}$ such that $f(x) \le c$ for every $x \in A$.

   (b) If $a > 1$, then $a^n \ge a$ for every $n \in \mathbb{N}$.

   (c) For every $\varepsilon > 0$, there exists a positive integer n^* such that $|a_n - A| < \varepsilon$ for all $n \ge n^*$.

4. Prove the following: for every $x \in [0, \pi/2]$, $\sin(x) + \cos(x) \ge 1$.