MAA 4102, MAA 5104
Homework 6
Due: Monday, February 20, 2017

Solve all problems and be sure to show all work. Answers with no supporting work will be given no credit.

1. Determine whether the given sequence $\{a_n\}$ converges or diverges with a_n as given. Be sure to prove your conclusion.
   (a) (p.72 2.1.2(b)) $a_n = \dfrac{n}{n^2-2}$
   (b) (p.72 2.1.2(g)) $a_n = \sqrt{n+1} - \sqrt{n}$
   (c) (p.73 2.1.2(j)) $a_n = \begin{cases} 0 & \text{if } n \text{ is odd} \\ \frac{1}{n} & \text{if } n \text{ is even} \end{cases}$
   (d) $a_n = \sin(n!\pi q)$, for any $q \in \mathbb{Q}$.

2. A sequence $\{a_n\}$ is equivalent to a function $f : \mathbb{N} \to \mathbb{R}$ given by $f(n) := a_n$. Therefore, the notions of (strictly) increasing/decreasing can be defined for sequences just as for functions. Either give an example of a sequence or prove no such sequence can exist for the following:
   (a) convergent but neither increasing or decreasing,
   (b) convergent but not bounded,
   (c) divergent to $+\infty$ but not increasing,
   (d) unbounded but neither increasing or decreasing.

3. Suppose that $\{a_n\}$ is a sequence of integers (meaning $a_n \in \mathbb{Z}$ for all n) converging to A. Prove that $\{a_n\}$ is eventually constant, that is, $a_n = A$ for large enough n.

4. (p.73 2.1.5) Prove that if the sequence $\{a_n\}$ converges to $|A|$, then the sequence $\{|a_n|\}$ converges to $|A|$. Is the converse true?

5. (Try your best on this problem!) Let x be a real number in the interval $(0, 1)$. Consider the sequence $\{a_n\}$ given by deleting the first n digits of the decimal expansion of x. For example, if $x = 0.6283547\ldots$, then $a_1 = 0.283547\ldots$, $a_2 = 0.83547\ldots$, $a_3 = 0.3547\ldots$, etc. For what x does this sequence converge and what is the limit?