

MAA 4102, MAA 5104
Homework 7
Due: Friday, February 24, 2017

Solve all problems and be sure to show all work. Answers with no supporting work will be given no credit.

1. Determine whether the given limits exist and find their values.

(a) (2.2.11 (d)) For $r \in \mathbb{R}$, $\lim_{n \rightarrow \infty} \frac{r^n}{n!}$,

(b) (2.2.11 (j)) $\lim_{n \rightarrow \infty} \sqrt[n]{2^{n+1}}$,

(c) (2.3.3 (b)) $\lim_{n \rightarrow \infty} \frac{n^3 - n + 1}{2n + 4}$.

2. Construct examples of sequences $\{a_n\}$ and $\{b_n\}$ such that $\{a_n\}$ diverges to $+\infty$, $\{b_n\}$ diverges to $-\infty$, and:

(a) $\{a_n + b_n\}$ diverges to $+\infty$,

(b) $\{a_n + b_n\}$ converges to c , for any $c \in \mathbb{R}$,

(c) $\{a_n + b_n\}$ diverges to $-\infty$, or

(d) none of these.

3. Show that $\{a_n\}$ converges to 0, where

$$a_n = \frac{\sin(3 + \pi^{n^2})}{n^{3/2}}.$$

4. Suppose that $\{a_n\}$ is a bounded sequence. Consider the sequence $\{x_n\}$, where

$$x_n = \sup\{a_k \mid k > n\}.$$

Is $\{x_n\}$ bounded? Is $\{x_n\}$ convergent? Is $\{x_n\}$ monotone or eventually monotone?