

**MAA 4102, MAA 5104**  
**Homework 8**  
**Due: Friday, March 3, 2017**

Solve all problems and be sure to show all work. Answers with no supporting work will be given no credit.

1. Without using any theorems, prove that  $\{a_n\}$ , with

$$a_n = \frac{2n + 3}{n^3 + 1},$$

converges to 0.

2. Let  $f : \mathbb{N} \rightarrow \mathbb{R}$  be any function. Determine whether the following limit exists and find its value.

$$\lim_{n \rightarrow \infty} \frac{\cos(f(n))}{n^p},$$

where  $p > 0$  is a constant.

3. Without using any theorems, prove that  $\{a_n\}$ , with

$$a_n = \frac{n^3 - 2n + 7}{3n + 2}$$

diverges to  $+\infty$ .

4. Determine the limiting value of each sequence, provided it exists. Prove your conclusions.

(a)  $a_1 = \sqrt{2}$  and  $a_{n+1} = \sqrt{2a_n}$  for all  $n \in \mathbb{N}$

(b)  $b_1 = 1$  and  $b_{n+1} = 3b_n - 1$  for all  $n \in \mathbb{N}$ .