MAA 4102, MAA 5104
Homework 9
Due: Friday, March 24, 2017

Solve all problems and be sure to show all work. Answers with no supporting work will be given no credit.

1. (p.103 2.5.2) Find the set of all accumulation points for the given set S.
   (a) $S = \{x \mid 0 < |x - 2| < 3\}$
   (b) $S = \{z \mid z \in [0, 1] \cap \mathbb{Q}\}$
   (c) $S = \{w \mid w \in (5, \infty)\}$
2. (p.104 2.5.7) Determine which of the following sequences are Cauchy. Verify your answer without using Theorem 2.5.9.
   (a) $a_n = \frac{n+1}{n}$
   (b) $b_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$
   (c) $c_n = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2}$
3. (p.110 2.6.2) Verify that the following sequences diverge. Find all subsequential limits, $\limsup_{n\to\infty} a_n$, and $\liminf_{n\to\infty} a_n$.
   (a) $a_n = \frac{4+(-1)^{n+1}}{3}$
   (b) $a_n = r^n$, with $r \leq -1$ or $r > 1$
   (c) $a_n = \cos\left(\frac{(n+1)\pi}{2}\right)$
4. (p.110 2.6.5) Prove **one** of the following statements.
   - Every unbounded sequence contains a monotone subsequence that diverges to infinity.
   - For any accumulation point s_0 of $\{a_n \mid n \in \mathbb{N}\}$, there exists a subsequence of $\{a_n\}$ converging to s_0.
5. (p.124 3.1.5) Determine whether or not the given limits exist. Find the values for those that do. Prove your assertion using any definitions or theorems from 3.1.
   (a) $\lim_{x\to\infty} \frac{3x}{x^2 - \sqrt{2}x}$
   (b) $\lim_{x\to\infty} \sqrt{x}$
   (c) $\lim_{x\to\infty} \frac{x-1}{|x-1|}$