1 Homework Set Three

Due Wednesday, February 12

Exercise 1.1. Give a proof that $A \rightarrow B \vdash \neg B \rightarrow \neg A$

Exercise 1.2. Give a proof that $A \land (B \lor C) \vdash (A \land B) \lor (A \land C)$

Exercise 1.3. Give the case for $\rightarrow$ - Introduction in the proof of the Compactness Theorem 1.4.8.

Exercise 1.4. Give the case for $\lor$ - Application (Elimination) in the proof of the Soundness Theorem.

Exercise 1.5. Give the case for the connective $\land$ in the proof of Proposition 1.6.9.
2 Homework Set Two

Due Wednesday, January 29

Exercise 2.1. Use truth tables to show \( \{(A \rightarrow B), (B \rightarrow C)\} \models (A \rightarrow C) \).

Exercise 2.2. Use truth tables to show that \( \{(A \lor B), (B \lor C)\} \) does not logically imply \( (A \lor C) \).

Exercise 2.3. Investigate the following sets of formulas for satisfiability. For those that are satisfiable, give an interpretation which makes them all true. For those that are not satisfiable, give a brief explanation.

(a) \( \{(A \lor B) \rightarrow (C \land D), (D \lor E) \rightarrow G, A \lor \neg G\} \)

(b) \( \{(A \rightarrow B) \land C, (D \rightarrow B) \land E), G \rightarrow \neg A, H \rightarrow I, \neg (\neg C \rightarrow E)\} \)

Exercise 2.4. The majority sentence \( \phi(A, B, C) \) is true if at least two of \( A, B, C \) are true and thus is false if at least two of the three are false. Construct a propositional sentence with this property.

Exercise 2.5. Show that the binary connective \( A \| B \), meaning “Not both \( A \) and \( B \)”, suffices to define negation and the usual binary connectives (and hence any possible sentence).
3 Homework Set One

Due Friday, January 17

Exercise 3.1. Determine which of the following strings is a sentence, where $A_0, A_1, A_2$ are propositional variables.

(a) $(A_0 \rightarrow (\neg A_1 \lor (\neg A_2)))$

(b) $(A_1 \rightarrow (A_1 \neg \land A_2))$

Exercise 3.2. Find a derivation of the sentence

$$B = (\neg((A_0 \land (A_1 \lor (\neg A_2))) \rightarrow A_3))$$

and use this to calculate the rank of $B$.

Exercise 3.3. Prove by induction on sentences that in any sentence $P$, the number of occurrences of propositional variables equals one plus the number of occurrences of binary connectives.

Exercise 3.4. Put the parentheses back into the simplified sentence

$$A \lor \neg B \rightarrow C \land D$$

to make it a legal sentence.