

Two-to-One Structures

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Abstract

We investigate computability-theoretic properties of computable structures with single unary functions f such that, for every x in the image, $f^{-1}(x)$ has exactly two elements, which we call 2:1 structures. We also investigate structures for which $f^{-1}(x)$ has either exactly two or zero elements, which we call (2,0):1 structures. In particular, we are interested in the complexity of isomorphisms between these structures. We prove that a computable 2:1 structure \mathcal{A} is computably categorical if and only if \mathcal{A} has only finitely many \mathbb{Z} -chains. We show that every computable 2:1 structure is Δ_2^0 -categorical. We further investigate computable and higher level categoricity of various natural subclasses of (2,0):1 structures, including highly computable and locally finite structures.