Effective Categoricity of Injection Structures

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Abstract

We study computability theoretic properties of computable injection structures and the complexity of isomorphisms between these structures. We prove that a computable injection structure is computably categorical if and only if it has finitely many infinite orbits. We also prove that a computable injection structure is Δ^0_2 categorical if and only if it has finitely many orbits of type ω or finitely many orbits of type Z. Furthermore, every computably categorical injection structure is relatively computably categorical, and every Δ^0_2 categorical injection structure is relatively Δ^0_2 categorical. We investigate analogues of these results for Σ^0_1, Π^0_1, and n-c.e. injection structures.

We study the complexity of the set of elements with orbits of a given type in computable injection structures. For example, we show that for every c.e. Turing degree $\mathbf{b}$, there is a computable injection structure $\mathcal{A}$ in which the set of all elements with finite orbits has degree $\mathbf{b}$ and, for every Σ^0_2 Turing degree $\mathbf{c}$, there is a computable injection structure $\mathcal{B}$ in which the set of elements with orbits of type ω has degree $\mathbf{c}$. We also study various index set results for infinite computable injection structures. For example, we show that the index set of infinite computably categorical injection structures is a Σ^0_3 complete set and that the index set of infinite Δ^0_2 categorical injection structure is a Σ^0_4 complete set.

We also explore the connection between the complexity of the character and the first-order theory of computable injection structures. We show that for an injection structure with a computable character, there is a decidable structure isomorphic to it. However, there are computably categorical injection structures with undecidable theories.