Injection structures specified by finite state transducers

Sam Buss[1], Douglas Cenzer[2], Mia Minnes[3] and Jeffrey B. Remmel[1]

[1] Department of Mathematics, University of California-San Diego, La Jolla, CA 92093-0112
sbuss@ucsd.edu, jremmel@ucsd.edu
[2] Department of Mathematics, University of Florida, Gainesville, FL 32611 *cenzer@math.ufl.edu,*
[3] Department of Computer Science and Engineering, University of California-San Diego, La Jolla, CA 92093-0404
minnes@eng.ucsd.edu

Abstract. An injection structure $\mathcal{A} = (A, f)$ is a set A together with a one-place one-to-one function f. $\mathcal{A}$ is an FST injection structure if A is a regular set, that is, the set of words accepted by some finite automaton, and f is realized by a finite-state transducer. We initiate the study of FST injection structures. We show that the model checking problem for FST injection structures is undecidable which contrasts with the fact that the model checking problem for automatic relational structures is decidable. We also explore which isomorphisms types of injection structures can be realized by FST injections. For example, we completely characterize the isomorphism types that can be realized by FST injection structures over a unary alphabet. We show that any FST injection structure is isomorphic to an FST injection structure over a binary alphabet. We also prove a number of positive and negative results about the possible isomorphism types of FST injection structures over an arbitrary alphabet.
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