Online Computability and Differentiation in the Cantor Space

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Abstract. This paper investigates a notion of differentiation for functions on the Cantor space. We study the existence and complexity of this derivative, particularly for online and computable online functions. It is shown that a random online function has no derivative at any computable point. It is shown that if a computable online function F has derivate $m > 0$ at a weakly 1-random point, then F has derivative m on a set of positive measure. We also explore the family of online functions on the Cantor space which represent real-valued functions.