

Effective Categoricity of Automatic Equivalence and Nested Equivalence Structures

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the date of receipt and acceptance should be inserted later

Abstract We study automatic equivalence and nested equivalence structures. The goal is to compare and contrast these automatic structures with computable equivalence and nested equivalence structures. Equivalence structures \mathcal{A} may be characterized by their characters $\chi_{\mathcal{A}}$ which encodes the number of equivalence classes of any given size. The characters of computably categorical, Δ_2^0 categorical but not computably categorical, or Δ_3^0 categorical but not Δ_2^0 categorical have been determined. We show that every computably categorical equivalence structure has an automatic copy, but not every Δ_2^0 categorical structure has an automatic copy. We construct an automatic equivalence structure which is Δ_2^0 categorical but not computably categorical and another automatic equivalence structure which is not Δ_2^0 categorical. We observe that the theory of an automatic equivalence structure is decidable and hence the character of any automatic equivalence structure is computable. On the other hand, there is a computable character which is not the character of any automatic equivalence structure. We show that any two automatic equivalence structures which are isomorphic are in fact computably isomorphic. Moreover, we show that for certain characters, there is always a double exponential time isomorphism between two automatic equivalence structures with that character. Finally, we briefly consider nested equivalence structures and construct an automatic nested equivalence structure that is not Δ_3^0 categorical but Δ_4^0 categorical and an automatic nested equivalence structure that is not Δ_4^0 categorical.