DENSELY COMPUTABLE STRUCTURES

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Abstract. In recent years, computability theorists have extensively studied generically and coarsely computable sets. This study of approximate computability was originally motivated by asymptotic density problems in combinatorial group theory. We generalize the notions of generic and coarse computability of sets, introduced by Jockusch and Schupp, to arbitrary structures by defining generically and coarsely computable and computably enumerable structures. There are two directions in which these notions could potentially trivialize: either all structures could have a densely computable copy, or only those having a computable (or computably enumerable) copy. We show that some particular classes of structures realize each of these extremal conditions, while other classes realize neither of them.

To further explore these concepts, we introduce a graded family of elementarity conditions for substructures, in which we require that the dense sets under consideration be "strong" substructures of the original structure. Here, again, for a given class, the notion could trivialize in the same two directions and we show that both are possible. For each class that we investigate, there is some natural number n such that requiring Σ_n elementarity of substructures is enough to trivialize the class of generically or densely computable structures, witnessing the essentially structural character of these notions.

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