Algorithmic Randomness of Continuous Functions

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Abstract

We investigate notions of randomness in the space $\mathcal{C}[2^{\mathbb{N}}]$ of nonempty closed subsets of $\{0,1\}^{\mathbb{N}}$. A probability measure is given and a version of the Martin-Löf test for randomness is defined. Π^0_2 random closed sets exist but there are no random Π^0_1 closed sets. It is shown that a random closed set is perfect, has measure 0, and has box dimension $\log_2 \frac{4}{3}$. A random closed set has no n-c.e. elements. A closed subset of $2^{\mathbb{N}}$ may be defined as the set of infinite paths through a tree and so the problem of compressibility of trees is explored. If $T_n = T \cap \{0,1\}^n$, then for any random closed set $[T]$ where T has no dead ends, $K(T_n) \geq n - O(1)$ but for any k, $K(T_n) \leq 2^{n-k} + O(1)$, where $K(\sigma)$ is the prefix-free complexity of $\sigma \in \{0,1\}^*$.