Algorithmic Randomness of Continuous Functions

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Abstract

We investigate notions of randomness in the space $\mathcal{C}(2^{\mathbb{N}})$ of continuous functions on $2^{\mathbb{N}}$. A probability measure is given and a version of the Martin-Löf Test for randomness is defined. Random Δ_2^0 continuous functions exist, but no computable function can be random and no random function can map a computable real to a computable real. The image of a random continuous function is always a perfect set and hence uncountable. For any $y \in 2^{\mathbb{N}}$, there exists a random continuous function F with y in the image of F. Thus the image of a random continuous function need not be a random closed set. The set of zeroes of a random continuous function is always a random closed set.