Effectively Closed Sets and Enumerations

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March 28, 2008

Abstract

An effectively closed set, or Π^0_1 class, may viewed as the set of infinite paths through a computable tree. A *numbering*, or *enumeration*, is a map from ω onto a countable collection of objects. One numbering is reducible to another if equality holds after the second is composed with a computable function. Many commonly used numberings of Π^0_1 classes are shown to be mutually reducible via a computable permutation. Computable injective numberings are given for the family of Π^0_1 classes and for the subclasses of decidable and of homogeneous Π^0_1 classes. However no computable numberings exist for small or thin classes. No computable numbering of trees exists that includes all computable trees without dead ends.