Degrees of difficulty of generalized r.e. separating classes

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Abstract

Important examples of Π^0_1 classes of functions $f \in {}^\omega\omega$ are the classes of *sets* (elements of ${}^\omega 2$) which separate a given pair of disjoint r.e. sets: $\mathsf{S}_2(A_0, A_1) := \{ f \in {}^\omega 2 : (\forall i < 2)(\forall x \in A_i) f(x) \neq i \}$. A wider class consists of the classes of functions $f \in {}^\omega k$ which in a generalized sense separate a k-tuple of r.e. sets (not necessarily pairwise disjoint) for each $k \in \omega$: $\mathsf{S}_k(A_0, \ldots, A_{k-1}) := \{ f \in {}^\omega k : (\forall i < k)(\forall x \in A_i) f(x) \neq i \}$. We study the structure of the Medvedev degrees of such classes and show that the set of degrees realized depends strongly on both k and the extent to which the r.e. sets intersect. Let $\mathcal{S}^m_k$ denote the Medvedev degrees of those $\mathsf{S}_k(A_0, \ldots, A_{k-1})$ such that no $m+1$ sets among $A_0, \ldots, A_{k-1}$ have a nonempty intersection. It is shown that each $\mathcal{S}^m_k$ is an upper semi-lattice but not a lattice. The degree of the set of k-ary diagonally nonrecursive functions DNR_k is the greatest element of $\mathcal{S}^1_k$. If $2 \leq l < k$, then $\mathbf{0}_M$ is the only degree in $\mathcal{S}^1_l$ which is below a member of $\mathcal{S}^1_k$. Each $\mathcal{S}^m_k$ is densely ordered and has the splitting property and the same holds for the lattice $\mathcal{L}^m_k$ it generates. The elements of $\mathcal{S}^m_k$ are exactly the joins of elements of $\mathcal{S}^1_i$ for $\lceil \frac{k}{m} \rceil \leq i \leq k$.