Immunity and Non-Cupping for Closed Sets

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Abstract

We extend the notion of immunity to closed sets and to Π^0_1 classes in particular in two ways: *immunity* meaning the corresponding tree has no infinite computable subset, and *tree-immunity* meaning it has no infinite computable sub*tree*. We separate these notions from each other and that of being *special*, and show separating classes for computably inseparable c.e. sets are immune and perfect thin classes are tree-immune. We define the notion of *prompt immunity* and construct a positive-measure promptly immune Π^0_1 class. We show that no immune-free Π^0_1 class P *cups* to the Medvedev complete class DNC of diagonally noncomputable sets, where P cups to Q in the Medvedv degrees of Π^0_1 classes if there is a class R such that the product $P \otimes R \equiv_M Q$. We characterize the interaction between (tree-)immunity and Medvedev meet and join, showing the (tree-)immune degrees form prime ideals in the Medvedev lattice. We show that every random closed set is immune and not small, and every small special class is immune.