Index Sets for Finite Normal Predicate Logic Programs

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Abstract

Let L be a computable first order predicate language with infinitely many constant symbols and infinitely many n-ary predicate symbols and n-ary function symbols for all $n \geq 1$ and let $Q_0, Q_1, \ldots$ be an effective list of all the finite normal predicate logic programs over L. Given some property P of finite normal predicate logic programs over L, we define the index set I_P to be the set of indices e such that Q_e has property P. Let $T_0, T_1, \ldots$ be an effective list of all primitive recursive trees contained in the finite sequences of integers. Then $[T_0], [T_1], \ldots$ is an effective list of all Π^0_1 classes where for any tree T, $[T]$ denotes the set of infinite paths through T. We modify constructions of Marek, Nerode, and Remmel to construct recursive functions f and g such that for all indices e, (i) there is a one-to-one degree preserving correspondence between the set of stable models of Q_e and the set of infinite paths through $T_{f(e)}$ and (ii) there is a one-to-one degree preserving correspondence between the set of infinite paths through T_e and the set of stable models $Q_{g(e)}$. We use these two recursive functions to reduce the problem of finding the complexity of the index set I_P for various properties P of normal finite predicate logic programs to the problem of computing index sets for primitive recursive trees for which there is a large variety of results.

For example, we use our correspondences to determine the complexity of the index sets relative to all finite predicate logic programs and relative to certain special classes of finite predicate logic programs of properties such as (i) having no stable models, (ii) having at least one stable model, (iii) having exactly c stable models for any given positive integer c, (iv) having only finitely many stable models, or (vi) having infinitely many stable models.