The Workshop on Model Theory and Computable Model Theory took place February 5-10, 2007 at the University of Florida in Gainesville, Florida as part of the National Science Foundation-sponsored Special Year in Logic. This special issue consists of selected papers from the conference. The workshop brought together researchers in model theory and researchers in computable model theory, areas which have diverged in recent years.

Model theory studies the relationship between general properties of structures and their theories and definable subsets of their universes. An introduction to the subject is given by Marker [20]. Computable model theory investigates the relationship between computability theoretic properties of countable structures and their theories and definable sets. Thus, computable model theory and computable algebra include the study of the computability of structures, substructures, isomorphisms and theories. A set Γ of sentences, such as a theory, is said to be *decidable* if it is computable, that is, if there is an algorithm for deciding whether a sentence is in Γ. A structure $\mathcal{M}$ the domain of which is ω is *computable* if its atomic diagram $A(\mathcal{M})$ is decidable, and $\mathcal{M}$ is *decidable* if its elementary diagram $T(\mathcal{M})$ is decidable.

By effectivizing Henkin's construction, we can show that every consistent decidable theory has a decidable model. For a decidable uncountably categorical theory T, Harrington and Khisamiev (independently) showed that every countable model of T is isomorphic to a decidable model. For an uncountably categorical theory that is not decidable, it is possible that some of the countable models are isomorphic to computable ones, while others are not. Goncharov et al. [14] showed that trivial, strongly minimal theories are model complete after naming constants for a model, and hence are $\forall\exists$-axiomatizable. This implies that if a trivial, strongly minimal theory has a computable model, then all of its countable models are isomorphic to $\mathbf{0}''$-decidable models. Khoussainov et al. [19] later showed that this is the best possible bound for these theories. Furthermore, Dolich et al.[7] established model completeness for every trivial uncountably categorical theory of Morley Rank 1, after naming constants for a model. Previously, Marker [21] constructed a trivial totally categorical theory of Morley Rank 2, which is not model complete after naming any set of constants.

Michael Laskowski [The elementary diagram of a trivial, weakly minimal structure is near model complete] continues this line of research on bounded quantifier depth of the elementary diagram of any model of the theory. He proves that if $\mathcal{M}$ is any model of a trivial, weakly minimal theory, then the elementary diagram $T(\mathcal{M})$ eliminates quantifiers down to Boolean combinations of certain existential formulas. A trivial, weakly minimal theory has a well behaved forking notion defined by algebraic closure. It does not have the finite cover property. The existential formulas used in the quantifier elimination are obtained from a class of quantifier-free, mutual algebraic formulas $\psi(\overrightarrow{z})$ by partitioning into $\overrightarrow{z}$ into $\overrightarrow{z} = \overrightarrow{x}\,\hat{}\,\overrightarrow{y}$ and existentially quantifying over $\overrightarrow{x}$.

In 1995, Pillay conjectured that all supersimple fields are perfect, pseudo algebraically closed (PAC), and with bounded absolute Galois group, that is, have finitely many open subgroups of index n for every n. In [23], Pillay and Poizat showed that supersimple fields are perfect and have bounded absolute

Galois group. A perfect fields K is PAC if every absolutely irreducible plane curve over K has a K-rational point.

Amador Martin-Pizarro and F.O. Wagner [Supersimplicity and Quadratic Extensions] prove that if K is a supersimple field with exactly one extension of degree 2 (up to isomorphism), then any elliptic curve E defined over K has an s-generic K-rational point, that is, a point $P \in E(K)$ such that $SU(P/F) = SU(K)$, where F is some small set of parameters over which E is defined. The importance of this theorem is that it holds for all elliptic curves. It uses the group law in the elliptic curves and it is not clear how to handle the curves of larger genus. It also remains open to generalize the result to fields with any number of extensions of degree 2.

Computable model theory and algebra includes the study of the computability of structures, substructures, isomorphisms and theories. A set Γ of sentences, such as a theory, is said to be *decidable* if it is computable, that is, if there is an uniform algorithm for deciding the sentences from Γ.

Interest in the questions of decidability and existential definability goes back to Hilbert's Tenth Problem (HTP) of whether there is a uniform algorithm to determine whether a given polynomial in several variables over $\mathbb{Z}$ has solutions in $\mathbb{Z}$. This has been answered negatively in the work of M. Davis, H. Putnam, J. Robinson and Yu. Matijasevich [12]. Following this work, similar questions have been raised for other fields and rings. That is, given a computable ring R, is there a uniform algorithm to determine whether an arbitrary polynomial in several variables over R has solutions in R? One way to obtain a negative solution of this question over a ring R of characteristic zero is to construct a Diophantine definition of $\mathbb{Z}$ over R. Using norm equations, Diophantine definitions have been obtained for $\mathbb{Z}$ over the rings of algebraic integers of various number fields [16] and also over certain "large" subrings of totally real number fields [27]. Another method of constructing Diophantine definitions [6] uses elliptic curves. If K is a totally real algebraic extension of $\mathbb{Q}$ and there exists an elliptic curve E over $\mathbb{Q}$ such that $[E(K) : E(Q)] < \infty$, then $\mathbb{Z}$ has a Diophantine defintion over $\mathcal{O}_K$. A survey of the area is given by Shlapentokh [26].

Alexandra Shlapentokh [Rings of algebraic numbers in infinite extensions of Q and elliptic curves retaining their rank] shows that elliptic curves whose Mordell-Weil groups are finitely generated over some infinite extensions of $\mathbb{Q}$ can be used to show the Diophantine undecidability of the rings of integers and bigger rings contained in some infinite extensions of rational numbers. In particular, let K be a totally real possibly infinite extension of $\mathbb{Q}$ and let U be a finite extension of K such that there is an elliptic curve E defined over U with $E(U)$ finitely generated and of positive rank. Then $\mathbb{Z}$ is existentially definable and HTP is unsolvable over the ring of integers of K.

We say that computable structures $\mathcal{A}_1$ and $\mathcal{A}_2$ have the same computable isomorphism type if there is a computable isomorphism between them. Existence of an isomorphism between computable structures does not always imply that there is a computable isomorphism between them. Let $\mathcal{A}$ be a structure. If $\mathcal{B}$ is computable and is isomorphic to $\mathcal{A}$, then $\mathcal{B}$ is called a computable presentation (or copy) of $\mathcal{A}$. The number of computable isomorphism types of $\mathcal{A}$, denoted by $\dim(\mathcal{A})$, is called the *computable dimension* of $\mathcal{A}$. It is obvious that $\dim(\mathcal{A}) = 1$ if and only if any two computable presentations of $\mathcal{A}$ are computably isomorphic. In case $\dim(\mathcal{A}) = 1$, then we say that $\mathcal{A}$ is computably categorical.

One of the central topics in computable model theory is the study of computable dimensions of structures and characterizations of computable categoricity. Here we provide several examples. Goncharov proved that for any $n \in \omega \cup \{\omega\}$ there exists a structure of computable dimension n [13]. In [5] Cholak, Goncharov, Khoussainov, Shore gave an example of a computably categorical structure $\mathcal{A}$ such that for each $a \in A$ the structure $(\mathcal{A}, a)$ has computable dimension n, where $n \in \omega$. Goncharov and Remmel proved that a linearly ordered set is computably categorical if and only if the set of successive pairs in the order is finite [25, 11]. Calvert, Cenzer, Harizanov and Morozov [1] showed that an equivalence structure is computably categorical if and only if there is a bound b on the sizes of finite equivalence classes, and there is at most one $t \in \{1, ..., b\} \cup \{\omega\}$ with infinitely many classes of size t.

Wesley Calvert, Sergey Goncharov, Jessica Millar and Julia Knight [Categoricity of computable infinitary theories] answer a question posed by J. Millar and Sacks, on the categoricity of the computable infinitary theories of structures with Scott rank ω_1^{CK}. In previous work, various subsets of the authors had produced computable structures of various kinds (trees [3], undirected graphs, fields, linear orderings [2]) with Scott rank ω_1^{CK}. J. Millar and Sacks asked whether it was possible that a computable structure with Scott rank ω_1^{CK} could have a computable infinitary theory that was $\aleph_0$-categorical [22]. It is natural to ask whether for known examples of computable structures of Scott rank ω_1^{CK} the theories are $\aleph_0$-categorical. The present paper gives an affirmative answer for several of the known examples; in particular, trees, undirected graphs, fields, and linear orderings.

Valentina Harizanov, Carl Jockusch and Julia Knight [Chains and antichains in partial orderings] study the complexity of infinite chains and antichains in computable partial orderings. It follows from a result of Jockusch [17] that a computable partial ordering has either an infinite Δ_2^0 chain or an infinite Δ_2^0 antichain, or else both an infinite Π_2^0 chain and an infinite Π_2^0 antichain. Hermann [15] constructed a computable partial ordering with no infinite Σ_2^0 chain or antichain. The present paper shows that there is a computable partial ordering which has an infinite chain but none that is Σ_1^1 or Π_1^1, and also obtain an analogous result for antichains. On the other hand, every computable partial ordering which has an antichain must have an infinite chain that is the difference of two Π_1^1 sets. The main result is that there is a computably axiomatizable theory of partial orderings which has a computable model with arbitarily long finite chains but no computable model with an infinite chain, and similarly for antichains. It is shown that if a computable partial ordering $\mathcal{A}$ has the feature that for every $\mathcal{B} \cong \mathcal{A}$, there is an infinite chain or antichain which is Δ_2^0 relative to $\mathcal{B}$, then there is a uniform dichotomy: either every copy $\mathcal{B}$ of $\mathcal{A}$ has an infinite chain which is Δ_2^0 relative to $\mathcal{B}$, or every copy $\mathcal{B}$ of $\mathcal{A}$ has an infinite antichain which is Δ_2^0 relative to $\mathcal{B}$.

Jennifer Chubb, Valentina Harizanov, Andrey Frolov [Degree spectra of the successor relation of computable linear orderings] determine a condition ensuring the Turing degree spectrum of the successor relation of a linear ordering will be closed upward in the c.e. Turing degrees. The condition applies to a broad class of linear orderings, and those to which it does not apply are characterized. The *Turing degree spectrum* of a relation on a linear ordering is the class of Turing degrees of that relation in computable copies of the linear ordering. Surprisingly little is known about the degree spectrum of the successor relation in computable linear orderings. Of course, the successor relation is always intrinsically co-c.e., and it is intrinsically computable when it is finite. Downey and Moses [9] provide an example where it is intrinsically complete. Downey, Goncharov, and Hirschfeldt [10] ask whether the degree spectrum of the successor relation can consist of a single degree different from $\mathbf{0}$ and $\mathbf{0}'$, and a similar question for the degree spectrum of the atom relation of computable Boolean algebras with infinitely many atoms was resolved by Downey and Remmel. Remmel [24] established that such a spectrum is closed upward in the c.e. degrees, and Downey [8] showed that such a spectrum must contain an incomplete degree. The result in the present article provides that every upper cone of c.e. degrees is realized as the Turing degree spectrum of some computable linear ordering.

Douglas Cenzer, Barbara Csima and Bakhadyr Khoussainov [Linear orders with distinguished function symbol] study certain linear orders with a function on them, and discuss for which types of functions the resulting structure is or is not computably categorical. In [18] Khoussainov provided examples of structures of type $(\mathcal{A}, h)$ where h is a function from A to A, of computable dimension n with $n \in \omega$. In [28] Ventsov studied computable dimensions of $(L; \leq, P)$ where $(L; \leq)$ is a linearly ordered set and P is a unary predicate. This paper is a continuation of the above work with an emphasis on computable dimensions of linearly ordered sets with distinguished endomorphisms. Particular structures include computable copies of the rationals with a fixed-point free automorphism, and also ω with a non-decreasing function.

Douglas Cenzer, Rod Downey, Jeffrey Remmel and Zia Uddin [Space complexity of torsion-free Abelian groups] continue the study of complexity theoretic model theory and algebra developed by Nerode, Remmel and Cenzer; see the handbook article [4] for details. Much of the work of those authors focused on polynomial time models. The present paper develops the theory of $LOGSPACE$ structures and apply it to the study of $LOGSPACE$ Abelian groups. It is shown that all computable torsion Abelian groups have $LOGSPACE$ presentations and the authors show that the groups $\mathbb{Z}$, $\mathbb{Z}(p^\infty)$, and the additive group of the rationals have $LOGSPACE$ presentations over a standard universe such as the tally representation and the binary representation of the natural numbers. The effective categoricity of such groups is also studied. For example, conditions are given under which two isomorphic $LOGSPACE$ structures will have a linear space isomorphism.

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