Find the first 5 nonzero terms of a power series solution to

\[ y'' = (\cos x)y; \quad y(0) = 6; \quad y'(0) = 12. \]
Find a minimum value for the radius of convergence of a power series solution about $x_0 = 4$ to the differential equation

$$(x^2 - 7x + 6)y'' - xy' + e^x y = 0.$$
Use power series to find a solution to $xy' = (x + 1)y$.

Find the first four nonzero terms, find a recurrence relation.

Then try to find the general terms of the power series and a closed form for the solution, from the power series.

HINT: $a_0 = 0$. 
Let $y = \sum_n a_n x^n$ be a power series solution to the differential equation

$$xy'' + y = 0; \quad y(0) = 0, \quad y'(0) = 1.$$ 

Find the first five nonzero terms of the series.
Find the power series expansion for \( F(x) = \int \frac{\ln(x+1)}{x} \, dx \).

Give the first four terms as well as the general formula.
Find the Taylor polynomial $P_4$ for a solution to the differential equation

$$y'' = y' + y \cos x; \quad y(0) = 1; \quad y'(0) = 2.$$
Find the transfer $Y(s)$ and the solution $y(t)$ to the differential equation

$$y'' + 2y' + 5y = g(t); \quad y(0) = 0; \quad y'(0) = 0$$

Express $Y(s)$ in terms of $G(s)$ and use convolution to express $y(t)$ in terms of $g(t)$—write out the integral.
Find the convolution \( \sin t * \sec t \) and observe that this is a solution to the differential equation \( y'' + y = \sec t \).
Solve the differential equation

\[ y'' + 4y = 6\delta(t - 3) \]

with \( y(0) = 2 \) and \( y'(0) = 0 \).

Express the solution in terms of the step function \( u(t - 4) \) and also in two cases, where \( t < 4 \) and \( t \geq 4 \).
Use Laplace Transforms to solve the differential equation

\[ y' + 2y = 12u(t - 4); \quad y(0) = 20 \]
Find the Laplace Transform $F(s)$ when

$$f(t) = \begin{cases} 
9, & \text{if } t < 3 \\
9t^2, & \text{if } t \geq 3 
\end{cases}$$
Use Laplace Transforms and Partial Fractions to solve

\[ y'' + y = 2; \quad y(0) = 0; \quad y'(0) = 4. \]
Find the function $g(t)$ which has Laplace Transform

$$G(s) = \frac{6s + 40}{s^2 + 8s + 25}$$
Given that the Laplace Transform $F(s)$ of $f(t) = t^{\frac{1}{2}}$ is $F(s) = \frac{\sqrt{\pi}}{2} s^{-\frac{1}{2}}$.

Find the Laplace Transforms $G(s)$ and $H(s)$ for

(a) $g(t) = e^{3t} f(t) = t^{\frac{1}{2}} e^{3t}$ ;
(b) $h(t) = t f(t) = t^{\frac{3}{2}}$. 
Use the definition of the Laplace Transform to determine $\mathcal{L}\{f\}$ for

$$f(t) = \begin{cases}
1, & 0 < t < 3 \\
0, & t > 3
\end{cases}$$
Use the elimination method to solve the following system.

\[ x' = -3x + y; \quad y' = x - 3y \]
The motion of a spring is governed by the differential equation

\[ x'' + 4x' + 5x = \cos 2t. \]

Find the steady-state solution \( x_p \).
Find a second solution to the differential equation

\[ y'' - \tan x \ y' - \sec^2 x \ y = 0 \]

given that \( y_1 = \sec x \) is a solution.

– Use Abel’s Identity and Reduction of Order.
Given the homogeneous solution \( y_h = c_1 x + c_2 x^3 \),

Find a particular solution \( y_p \) using Variation of Parameters:

\[
y'' - \frac{3y'}{x} + \frac{3y}{x^2} = 2x
\]
Set up the particular solution $y_p$ for the following differential equations but do not evaluate the constants. HINT: You need to find $y_h$ to check for matching.

$$y'' - 3y' + 2y = x^2e^x + e^{2x} \cos 3x$$
Use the Method of Undetermined Coefficients to solve the following problem with initial values $y(0) = 0$ and $y'(0) = 5$.

$y'' + 5y' + 6y = 3t.$

(HINT: $y_h = c_1e^{-2t} + c_2e^{-3t}$.)
Find the solution of the differential equation

\[ y'' + 6y' + 13y = 0; \quad y(0) = 5; \quad y'(0) = 1. \]
Solve the linear second order differential equation

\[ y'' + 8y' + 15y = 0 \] with initial values \( y(0) = 2 \) and \( y'(0) = -2 \).
The population of alligators in Florida is currently estimated at one million. Suppose that the natural growth rate is 10%.

(a) Estimate the population in 10 years given exponential growth.

(b) Estimate the population in 10 years, given an annual “harvest” of 50,000 alligators.

(c) Find the limiting value for the alligator population if the equation is $P' = .1P - 2 \times 10^{-7}P^2$ (in millions)
A brine solution with concentration .05 kg/L flows at a constant rate of 6 L/min into a tank and the well-stirred solution flows out at the same rate. The tank initially 50 L of a brine solution containing .5 kg of salt. Find the mass $A(t)$ of salt in the tank after $t$ minutes and also find the concentration of the solution after $t$ minutes. Do these have limiting values?
Classwork Seven: NAME:

Solve (implicitly) the homogeneous equation

\[ \frac{dy}{dx} = \frac{3y^2 - 2xy}{2xy - x^2} \]

by first transforming to a separable equation. Simplify the answer.
Find the family of orthogonal trajectories to the curves $y = cx^2$. Sketch roughly.
Use the Test for Exactness to check the differential equation

\[(y^2 \sec x \tan x + 2)\, dx + (2y \sec x - 3y^2)\, dy = 0\]

Then integrate to find the general (implicit) solution.
Finally, solve the initial value problem when \(x_0 = 0\) and \(y_0 = 2\).
1. Find the general (explicit) solution of the linear differential equation

\[ \frac{dy}{dt} + \frac{3y}{t} = \frac{4}{t} \]

Then solve the initial value problem with \( x(1) = 5 \).
Use Euler’s method with step size $h = .2$ to approximate $y(.6)$ in the initial value problem $\frac{dy}{dx} = 2x + y \quad y(0) = 1$
1. Solve the differential equation \( y' = y(y - 2)(y - 4)^2 \) graphically:
   (a) Find the constant solutions \( y = c \).
   (b) Find the intervals where \( y' \) is positive/negative and sketch the phase line.
   (c) Find the limiting value \( L \) on each interval.
   (d) Roughly sketch solution curves \( y(0) = 1 \), \( y(0) = 3 \), and \( y(0) = 5 \).
1. (a) Verify that \( y = x + 1 + Ce^x \) is a solution of \( y' = -x + y \) for any \( C \).

(b) Find the value of \( C \) if \( g(0) = 4 \).