

Evaluate the following integrals using substitution

1.  $\int \frac{dx}{4x^2+1}$

2.  $\int_0^3 x\sqrt{x+1} dx$

Evaluate the following integrals using substitution

1.  $\int_0^{\pi/4} \cos^3 x \sin x \, dx.$

2.  $\int \frac{x^2+1}{x^3+3x+4} \, dx.$

1. A bacteria colony grows exponentially at a rate of  $k = .2$  (per hour) starting from a population  $P(0) = 100$ . Find a formula for the population  $P(t)$  at time  $t$ , find  $P(5)$  and find the doubling time for  $P(t)$ .

2. The half-life of floridium is 10 years. Find a formula for the amount  $F(t)$  remaining from 5 mg of floridium after  $t$  years, using 2 as the base.

Find the rate  $k$  of exponential decay, so that  $F(t)$  satisfies the equation  $F' = kF$ .

1. A population  $P(t)$  of insects increases at a rate of  $400 + 6t + .3t^2$ . Find a formula for  $P(t)$  given that  $P(0) = 100$ .

2. A particle moves in a straight line with velocity  $v(t) = 6 - 2t$ . Find the net distance traveled and the total distance traveled between  $t = 0$  and  $t = 5$ .

1. Find a formula for the function  $F(x) = \int_2^x 3t^2 dt$ .

2. Find  $G'(x)$ , where  $G(x) = \int_1^{3x} \frac{\sin t}{t} dt$ .

Hint: Let  $F(x) = \int_1^x \frac{\sin t}{t} dt$ , so  $G(x) = F(3x)$ .

Evaluate the definite integrals

1.  $\int_4^9 \sqrt{x} \, dx$

2.  $\int_0^{\ln 2} e^{2x} \, dx.$

3.  $\int_0^{\pi/4} \sec^2 x \, dx.$

1. Evaluate the integral by interpreting it in terms of areas:  $\int_0^5 (4 + \sqrt{25 - x^2}) dx$

2. Evaluate  $S_n = \sum_{i=1}^n \frac{3i^2 + 4in}{n^3}$  in general and for  $n = 10$ .  
Then find  $\lim_{n \rightarrow \infty} S_n$ .

1. Evaluate  $\sum_{n=1}^6 3n^2 + 2n$

2. Estimate the area under  $y = f(x) = \sin x$  from  $x = 0$  to  $x = \pi$  by approximation with 6 rectangles of equal width, using left endpoints  $x_i^* = x_{i-1}$ .



1. Evaluate the following indefinite integrals

(a)  $\int \frac{2x+3}{x}$

(b)  $\int \cos 2x + \sec^2 x.$

2. Find the antiderivative  $F(x)$  of  $f(x) = 8x - 6\sqrt{x}$  such that  $F(4) = 40$ .

Find the point on the hyperbola  $xy = 8$  that is closest to the point  $(3,0)$ .

Find the indeterminate form and use L'Hopital's Rule to evaluate the limits:

1.  $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x^2}$

2.  $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x$

Find all asymptotes, transition points, intervals where  $f$  is increasing/decreasing and concave up/down. Then sketch of the graph of  $f(x) = x - 10 + 16x^{-1}$ .

Let  $f(x) = x^3 - 3x^2 - 2$ .

Find the critical numbers and the intervals on which  $f$  is increasing / decreasing.

Use the second derivative test to see which is a local maximum or minimum.

Find the point of inflection and determine where  $f$  is concave up / down.

Let  $f(x) = x^3(x - 5)^2$ .

Find the critical numbers and determine where  $f$  is increasing and where  $f$  is decreasing.

Then use the first derivative test to determine whether these are local maxima or minima.



Find the linear approximation  $L(x)$  to the function  $y = f(x) = x^{\frac{1}{3}}$  near  $(27, 3)$  and use this to approximate  $28^{\frac{1}{3}}$ .



Use implicit differentiation to find  $y'$  and  $y''$  at the point  $(3, -2)$  on the curve  $4x^2 + 2xy + 3y^2 = 36$ .

A tray is 10 cm long with triangular cross section 5 cm across and 4 cm deep. If the tray is filling with liquid at a rate of 2 cc per minute, how fast is the depth changing when it is 2 cm deep?

HINT: if  $y = \text{width}$  and  $x = \text{depth}$ , then  $y = 5x/4$ ; find volume  $V$  in terms of  $x$ .

1. Find the derivative of  $y = g(x) = 2^{\cos x}$ .

2. Find the derivative of  $y = f(x) = \ln(x^2 + 1)$ .

1. Find the derivative of  $f(x) = \tan^{-1}(x^2)$ . Then find the equation for the tangent line to  $f(x)$  at  $x = 1$ .

2. Let  $g(x)$  be the inverse function of  $f(x) = e^x + 2x$ .  
Given that  $f(0) = 1$ , compute  $g'(1)$ .

Find the general formula for the derivative of  $y = f(x) = \sqrt{3 + e^x}$ .  
Then find  $f'(0)$  and an equation for the tangent line at  $x = 0$ .

Find the general formula for the derivative of  $y = f(x) = \sin^3 2x$ .  
Then find the values of  $x$  in  $[-\pi, \pi]$  such that  $f'(x) = 0$ .

1. Find the derivative of  $f(x) = x \cos x$ .

2. Find the first and second derivatives of  $g(x) = \frac{\sin x}{x}$ .

Find the first and second derivatives of the following functions.

1.  $f(x) = (x^2 + x)e^x$ .

2.  $g(x) = \frac{1}{x^2+1}$



Find how long it takes to stop and how far a car travels when braking from 60 mph to 0 with a velocity  $v = 60 - 10t$  and distance traveled  $s = 60t - 5t^2$  after  $t$  seconds.

Find the following derivatives. Show how the product and/or quotient rules are used.

1.  $f(x) = x^2 e^x$ .

2.  $g(x) = \frac{\sqrt{x}}{x+1}$ .

Compute the derivatives of the following functions, using the power and exponent rules:

1.  $f(x) = 3x^4 - 2x^5$ .

2.  $f(x) = \sqrt{x^3}$

3.  $f(x) = \frac{2}{x^3}$ .

Use limits to find a general formula for the derivative of  $f(x) = \sqrt{x+1}$  at  $x = a$ .

Let  $f(x) = x^2$ , so that  $f(3) = 9$ .

1. Compute the limit  $\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3}$ .

2. Observe that the answer to part (1) is the slope of the tangent line to the curve  $y = x^2$  at the point  $(3, 9)$ . Now find the equation of this tangent line.

1. Let  $f(x) = \frac{2x}{\sqrt{x^2+4x}}$ . Find the limits  $\lim_{x \rightarrow -\infty} f(x)$  and  $\lim_{x \rightarrow \infty} f(x)$  and the corresponding horizontal asymptotes.

2. Let  $f(x) = \frac{36-4x^2}{x^2-16x}$ . Find the limits  $\lim_{x \rightarrow -\infty} f(x)$  and  $\lim_{x \rightarrow \infty} f(x)$  and the corresponding horizontal asymptotes. Also find the vertical asymptotes.

1. Use the Squeeze Theorem to find  $\lim_{x \rightarrow 0} x \cos(e^{\frac{1}{x}})$

2. Evaluate  $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2-1}$

Find  $\lim_{x \rightarrow 0^-} f(x)$ ,  $\lim_{x \rightarrow 0^+} f(x)$ ,  $\lim_{x \rightarrow 0} f(x)$ ,  $\lim_{x \rightarrow 1^-} f(x)$ ,  $\lim_{x \rightarrow 1^+} f(x)$ ,  $\lim_{x \rightarrow 1} f(x)$ , where

$$f(x) = \begin{cases} \sin x, & \text{if } x < 0; \\ x^2, & \text{if } 0 \leq x \leq 1; \\ x + 1, & \text{if } x > 1. \end{cases}$$

Then determine whether  $f$  is continuous, left continuous or right continuous, at  $x = 0$  and/or at  $x = 1$ .



Evaluate the following limits, if possible:

1.  $\lim_{x \rightarrow 3} \frac{1}{x-3} - \frac{1}{x^2-5x+6}$ .

2.  $\lim_{x \rightarrow 2^+} \frac{\ln(x-2)}{x-3}$ .

1. Use the limit definition to verify that  $\lim_{x \rightarrow 3} 2x - 1 = 5$ . How close to 3 does  $x$  need to be so that  $2x - 1$  is within .1 of 5?

2. Find the (possibly infinite) limits  $\lim_{x \rightarrow 2^-} \frac{1}{x^2 - 4}$  and  $\lim_{x \rightarrow 2^+} \frac{1}{x^2 - 4}$ .

1. The following table illustrates the growth in the population infected with the flu virus over time. Find the average rate of change in the infected population during the intervals  $[12, 18]$ ,  $[15, 18]$ , and  $[17, 18]$ . Estimate the exact rate of change for day 18.

Days	0	10	12	15	17	18	—
Infected	0	15	23	36	45	50	—

2. The graph below shows the number of Twitter users in Gainesville in recent years. Find the average increase in the number of users between 2010 and 2014. Estimate the exact rate of change at time  $t = 2013$  based on the given tangent line.

1. Simplify  $9^{2/3} \cdot 3^{5/3}$ .

2. Solve  $2 \ln x + \ln\left(1 + \frac{1}{x^2}\right) = \ln 5$ .

1. Find  $\sin^{-1}(\sin(\frac{17\pi}{3}))$  without any calculations.

2. Let  $f(x) = 3x - 5$ . Find a formula for the inverse function  $f^{-1}$ .

1. Given that  $0 \leq \theta \leq \pi/2$  and that  $\tan \theta = 2/3$ , find  $\cos \theta$  and  $\sin \theta$ .

2. Find a value of  $\theta$  such that  $\cos \theta = \sin 2\theta$ .

1. Find the domain and range of  $f(t) = \sqrt{t-2}$ .

2. Find the composition functions  $f \circ g$  and  $g \circ f$  where  $f(t) = 2t + 3$  and  $g(x) = 1/x$ .

1. Find the slope of the line  $2x + 3y = 5$ .

Hint: solve for  $y$ .

2. Find an equation of the line passing through points  $(2, 3)$  and  $(4, 8)$ .

3. Complete the square and find the minimum value of  $y = f(x) = 3x^2 + 12x - 5$ .



1. Express the set of numbers  $\{x : |2x - 6| < 5\}$  as an interval.  
Hint: first simplify the inequality  $|2x - 6| < 5$ .

2. Which function is odd/even/neither:

- a.  $f(x) = x^2 - 3$

- b.  $g(x) = x^4 + x^3$

- c.  $h(x) = \frac{1}{x^3 + 2x}$