Prove by induction that for \( n \geq 1 \),

\[
\frac{1}{2} + \frac{1}{6} + \cdots + \frac{1}{n(n+1)} = 1 - \frac{1}{n(n+1)}.
\]

Hint: \( \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \).
Prove by induction that for reals $x$ and all natural numbers $m, n$:

$$(x^m)^n = x^{mn}.$$ 

Use the recursive definition:

$z^0 = 1$ and $z^{n+1} = z^n \cdot z$
Prove by induction that for $n \geq 1$,

$$1 + 3 + \cdots + 2n - 1 = \sum_{i=1}^{n} 2n - 1 = n^2.$$
Let \( f(x) = (x - 1)^3 + 8 \), let \( g(x) = 1 + (x - 8)^{1/3} \), and let \( h(x) = (x + 1)^{1/3} - 1 \).

Determine whether \( g \) or \( h \) is the inverse function to \( f \) by finding formulas for \( h \circ f \) and \( g \circ f \) and seeing which one reduces to the identity.
Let \( f : [0, \infty) \to [0, \infty) \) be defined by \( f(x) = \log(3x + 2) \).

Give a formula for the inverse function \( f^{-1}(x) \)

Find the range \( Ran(f) \).

Explain why \( f \) is one-to-one and onto.
Define the function $f : [4, \infty) \rightarrow [0, \infty)$ by $f(x) = \sqrt{x - 4}$.

Prove that $f$ is one-to-one.
Find whether the following functions from $\mathbb{R}$ to $\mathbb{R}$ one-to-one or onto:

(a) $f(x) = e^x$

(b) $g(x) = \sin x$

(c) $h(x) = x^3 - x$
Let \( f(x) = x^2 \) and let \( g(x) = x + 3 \).

Find formulas for \( f \circ g \) and for \( g \circ f \).

Find the range of \( f \circ g \) and the range of \( g \circ f \).
Let $B \subseteq A$ and define the relation $R$ on $\mathcal{P}(A)$ by

$$R(C, D) \iff B \cap C = B \cap D$$

Show that this is an equivalence relation.
The equivalence relation $C_5$ is defined on integers by $5|(x - y)$.

Find the partition induced by $C_5$ on the set

$A = \{n \in \mathbb{Z} : 1 \leq n \leq 20\} = \{1, 2, 3, \ldots, 20\}$.

Give the equivalence class $[3]$ of 3 under this equivalence relation on $A$. 
Let \( A = \{\{1\}; \{1, 2\}; \{1, 4\}; \{1, 2, 3\}; \{1, 2, 4\}\} \).

(a) Determine the minimal elements of \( A \), the greatest lower bound of \( A \), and the smallest element, if any.

(b) Determine the maximal elements of \( A \), the least upper bound of \( A \), and the greatest element, if any.
Show that if $R$ and $S$ are both transitive, then $R \cap S$ is also transitive.
Show that for any relation $R \subseteq A \times B$, $\text{Ran}(R^{-1}) = \text{Dom}(R)$. 
Define the relation $R(x, y)$ on $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \times \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ by $R(x, y) \iff x | y \& x \neq y$.

- List the elements of $R$.
- Find the domain and range of $R$.
- Find the elements of the composition $R \circ R$.
- Find the domain and range of $R \circ R$. 
Prove that $\lim_{x \to 4} \frac{5x^2 - 20x}{x - 4} = 20$. 
Show that for any real number $x$, there is a unique $y$ such that $3x + 2y = 5$. 
Prove that if $|x - 5| > 3$, then $x < 2$ or $x > 8$. 
Prove that if \((\forall x)[P(x) \rightarrow Q(x)]\) and \((\exists x)P(x)\), then \((\exists x)Q(x)\).
Prove that $A \setminus (B \cup C) \subseteq A \setminus B$. 
Prove that if $a|b$ and $c|d$, then $ac|bd$. 
Show that if $x > 3$, then $x^2 - x > 6$. 
Verify that \( \bigcap_{i \in I} (A_i \setminus B_i) = (\bigcap_{i \in I} A_i) \setminus (\bigcup_{i \in I} B_i) \)
by writing out in sentence form.
Show that $(\exists x)(P(x) \lor Q(x))$ is logically equivalent to $(\exists x)P(x) \lor (\exists x)Q(x)$

Use the rule that $(\forall x)(P(x) \land Q(x))$ is logically equivalent to $(\forall x)P(x) \land (\forall x)Q(x)$
Show that $P \rightarrow (Q \rightarrow R)$ is logically equivalent to $(P \land Q) \rightarrow R$. 
Make Venn diagrams for the sets \((A \cup B) \setminus C\) and \(A \cup (B \setminus C)\).

Decide which set is necessarily a subset of the other.

Give an example where the two sets are different.
Write one or two alternative definitions for the following set:

\{1, 4, 9, 16, \ldots \}
Use the laws from the text to simplify $(P \land Q) \lor (P \land \neg Q)$. 
Make a truth table for the formula $P \land (\neg P \lor Q)$.
Can you find a simpler formula which is equivalent to $P \land (\neg P \lor Q)$?

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