Sample Problem Solutions for Exam Three

1. Find the linear approximation to $f(x) = x^{\frac{1}{4}}$ near the point (16,2) and use use this to estimate $18^{\frac{1}{4}}$.

2. Find the absolute maximum and minimum of $f(x) = x + \cos x$ on $[0, \pi]$.

3. Given that $f(x) = \frac{2(x+1)^2}{x^2+1}$, $f'(x) = \frac{-4(x^2-1)}{(x^2+1)^2}$, and $f''(x) = \frac{8x(x^2-3)}{(x^2+1)^3}$, find the intercepts, asymptotes, critical and inflection points, and sketch the graph.
   Note that $\sqrt{3} \simeq 1.73$, $f(-\sqrt{3}) \simeq 0.27$, and $f(\sqrt{3}) \simeq 3.73$.

4. Let $f(x) = x - 3x^{2/3}$. Find the domain, intercepts, symmetry, asymptotes, critical numbers, inflection points, intervals where increasing/decreasing and concave up/down. Sketch the graph.

5. Evaluate the following limits:
   (a) $\lim_{x \to \infty} x^3/e^x$
   (b) $\lim_{x \to 0} \frac{\sin x - x}{x^3}$.
   (c) $\lim_{x \to 1} (x^2 - 1)^{x-1}$

6. A window has the shape of a rectangle with a semicircle on one end. Find the dimensions of the window with the largest area with a perimeter of 20 feet.

7. Find the point on the line $y = 2 - x$ which is closest to the point $(1, 0)$.

8. A ball is tossed into the air with initial velocity 24.5 m/sec and subject to gravity, which is $-9.8 m/sec^2$. Find equations for the velocity $v(t)$ and height $s(t)$. Give the maximum height and the impact velocity.

9. A point moves along the x-axis with acceleration given by $a(t) = -1/t^2$. If $v(1) = 3$, find the distance it will travel between $t = 1$ and $t = 3$ seconds.

10. Suppose that $f'(x) = x^{2/3} + \sec x \tan x$ and that $f(0) = 2$. Find $f(x)$.

11. Find the following antiderivatives:
   (a) $\int 7x^{4/3}$
   (b) $\int (\frac{6}{x^3} + \frac{4}{x})$
   (c) $\int 10e^{-5x}$
   (d) $\int \cos(2t + 1)$.