

Sample Problems for Exam Two

1. Find the derivative of $\cos x$ as a limit.
2. Find the equation of the normal line to the curve $y = f(x) = (x^2 - x)^3$ at the point where $x = 2$.
3. A particle moves according the law of motion $s(t) = 2t^3 - 21t^2 + 60t$. Find the velocity function, determine when the particle is at rest and when it is moving forward. Find the average velocity from time $t = 5$ to $t = 10$ and find the total distance traveled from $t = 0$ to $t = 10$. Find the acceleration function and determine when the velocity is increasing and decreasing.
4. Find $f'(\frac{\pi}{4})$ if $f(x) = \frac{x}{\sin x}$.
5. Doctors estimate a person's body surface area S (in meters squared) by the formula $S = \sqrt{hm}/60$, where h is height in cm and m is mass in kg. Calculate the rate of change of S with respect to mass if $h = 180$ is constant. Find the rate at $m = 60$ and at $m = 70$.
6. Find $F'(3)$ where $F(x) = (f \circ g)(x)$, $g(3) = 5$, $g'(3) = 3$, $f'(3) = 1$ and $f'(5) = 4$.
7. Find $f'(x)$ and $f''(x)$ where $f(x) = \cos^2(3x)$.
8. Find $f'(3)$ and $f''(3)$ where $y = f(x)$, $f(3) = 1$ and $xy^3 + xy = 6$.
9. Find $f'(x)$ and $f''(x)$ where $f(x) = \frac{e^x}{e^x + 1}$.
10. Find the second derivative of $f(x) = x^2 \ln x$.
11. Let $f(x) = x + \sqrt{x^3 + 1}$ (so $f(2) = 5$) and that $g(x)$ is the inverse function of $f(x)$. Find $g'(5)$.
12. Use Logarithmic Differentiation to find $f'(1)$ for $f(x) = e^{x^2} (x + 3)^2 / (2x - 1)$.
13. Find the tangent line to $y = \sin^{-1} x$ at $x = \frac{1}{2}$ and sketch with the curve.
14. A spherical balloon is inflated at a rate of 20 cc per second. How fast is the radius of the sphere increasing when the radius equals 5 cm.