Laplace Transforms

DEFINITION. $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = F(s)$.

THEOREM. $\mathcal{L}\{c_1 y_1 + c_2 y_2\} = c_1\mathcal{L}\{y_1\} + c_2\mathcal{L}\{y_2\}$.

THEOREM. $\mathcal{L}\{e^{at} f(t)\} = F(s-a)$.

THEOREM. (a) $\mathcal{L}\{f'(t)\} = sF(s) - f(0)$.
(b) $\mathcal{L}\{f''(t)\} = s\mathcal{L}\{f'\} - f'(0) = s^2F(s) - sf(0) - f'(0)$.

THEOREM. $\mathcal{L}\{tf(t)\} = -F'(s)$.

$\Gamma(p+1) = \int_0^\infty x^p e^{-x}\,dx$

THEOREM. $\mathcal{L}\{t^p\} = \Gamma(p+1)/s^{p+1}$.

FACT. $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

FACT. $\Gamma(p+1) = p\Gamma(p)$.

DEFINITION. The step function $u(t-c) = \begin{cases} 0, & \text{if } t < c \\ 1, & \text{if } t \geq c. \end{cases}$

THEOREM. $\mathcal{L}\{u(t-c)f(t-c)\} = e^{-sc}F(s)$.

DEFINITION. The convolution $f(t) * g(t) = \int_0^t f(t-u)g(u)du$.

THEOREM. $\mathcal{L}f(t) * g(t) = \mathcal{L}f(t) \cdot \mathcal{L}g(t)$.

FACT. $f(t) * g(t) = g(t) * f(t)$.

THEOREM. If $f(t)$ is periodic with period T, then

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)dt.$$

FACT. $\mathcal{L}\{\delta(t-c)\} = e^{-sc}$.

FACTS $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$; $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$.

$\mathcal{L}\{cos\,bt\} = \frac{s}{s^2+b^2}$; $\mathcal{L}\{sin\,bt\} = \frac{b}{s^2+b^2}$.

Power Series

$f(x) = \sum_{n=0}^\infty a_n(x-x_0)^n$, where $a_n = f^{(n)}(x_0)/n!$.

$f'(x) = \sum_n na_n(x-x_0)^{n-1}$, $f''(x) = \sum_n n(n-1)a_n(x-x_0)^{n-2}$, etc.

$e^x = \sum_n \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$

$cos\ x = \sum_n (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \cdots$

$sin\ x = \sum_n (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{6} + \frac{x^5}{120} + \cdots$

$\frac{1}{1-x} = \sum_n x^n = 1 + x + x^2 + \cdots$

$\frac{1}{1+x} = \sum_n (-1)^n x^n = 1 - x + x^2 - x^3 + \cdots$

$ln(1+x) = \sum_{n=1}^\infty (-1)^{n+1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$

$\frac{1}{1+x^2} = \sum_n (-1)^n x^{2n} = 1 - x^2 + x^4 - x^6 + \cdots$

$arctan\ x = \sum_{n=1}^\infty (-1)^{n+1} \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$