Differential Equations Final Exam NAME:

(7) 1. Find the integrating factor μ and solve explicitly the linear differential equation

$$dy/dx - \frac{2y}{x} = x^2\ cos\ x$$

(8) 2. Use the Test for Exactness to check the differential equation

$$(2x + y^3\ sec^2 x)dx + (1 + 3y^2 tan\ x)dy = 0$$

Then integrate to find the general (implicit) solution.

Finally, solve the initial value problem when $x_0 = \pi$ and $y_0 = 2$.

(8) 3. Find the steady-state solution y_p of a spring-mass system subject to the differential equation

$$y'' + 4y' + 20y = sin\ 2t.$$

(Hint: Use Undetermined Coefficients.)

(7) 4. Use Variation of Parameters to find a particular solution to

$$y'' + y = sec\ t$$

(8) 5. Use Laplace Transforms and Partial Fractions to solve

$$y'' + 4y = 8t; \qquad y(0) = 3; \quad y'(0) = 0.$$

(7) 6. Use Laplace transforms to solve the differential equation

$$y' + 5y = 10\delta(t-3); \qquad y(0) = 100$$

Express the solution using step functions.

(8) 7. Use any method to find the first four nonzero terms in the Taylor polynomial approximation for the initial value problem

$$x' + (sin\ t)x = 0; \quad x(0) = 1$$

(7) 8.Find the indicial equation and solve the Cauchy-Euler differential equation

$$x^2y'' + 7xy' + 8y = 0; \qquad y(1) = 3 \quad y'(1) = 5$$

with initial values