Sample Problems for Exam Two

1. Compute the Wronskian of $y_1 = x^2, y_2 = x^2 ln(x)$; are y_1, y_2 independent?

2. Solve $y'' - 4y' + 5y = 0$ with $y(0) = 3$ and $y'(0) = 9$.

3. Use Undetermined Coefficients to solve $y'' + 4y = 6x + 4cos\ 2x$.

4. Use Variation of Parameters to solve $y'' - 4y' = e^{3x}$.

5. Find the general solution of the Euler equation $x^2 y'' + xy' - 4y = x^2 + 1$.
HINT: $y_h = c_1 x^{-2} + c_2 x^2$.

6. A 16 pound weight is suspended from a spring with $k = 18$ pounds per foot.

(a) The weight is pulled down 3 inches from equilibrium and then struck upwards with initial speed 2 feet per second. Find the equation of motion and give the amplitude, period and phase shift. Sketch the solution.

(b) An outside force of $12sin(6t)$ is applied to the spring at equilibrium. Write the differential equation and solve for $x(t)$. Find the equation of motion. What phenomenon does this represent?

7. A mass of 4 grams is suspended from a spring with constant 16 dynes per centimeter. The mass is pulled down .5 cm and an external force of $12sin(t)$ is applied. Assuming a damping factor of $8v$ (dynes), write the differential equation and solve for $x(t)$. What is the steady state solution.

8. Solve the system $x' = x - 4y; \quad y' = x + y$

9. Factor the differential equation $y'' - 3y' + 2y = x$ into two first order equations and solve.

10. Use reduction of order (by the Wronskian and Abel's identity) to find a second solution to $xy'' + (1 - 2x)y' + (x - 1)y = 0$ given that $y = e^x$ is one solution.

11. Find the general solution of $y''' - 3y' + 2y = 0$.

12. What does it mean to say that y_1, y_2 and y_3 are independent? Show that x, $x^2 - 1$ and $x^2 - 4$ are independent using the definition.

13. Find the general solution of $y^{(viii)} - y^{(vii)} - y^{(iv)} + y^{(iii)} = 0$, given that $r^8 - r^7 - r^4 + r^3 = r^3(r - 1)^2(r + 1)(r^2 + 1)$.

14. Use Undetermined Coefficients to solve $y''' - y'' + 4y' - 4y = cosx$.