Sample Problems for Exam Three

1. State and prove the formulas for
   (a) \( \mathcal{L}\{e^{at}f(t)\} \)  
   (b) \( \mathcal{L}\{f'(t)\} \)  
   (c) \( \mathcal{L}\{tf(t)\} \)  
   (d) \( \mathcal{L}\{u(t-a)f(t-a)\} \)

2. Define the Gamma function \( \Gamma(p+1) \) and show that \( \mathcal{L}\{t^p\} = \Gamma(p+1)/s^{p+1} \).

3. Find the Laplace transform \( F(s) \) of \( f(t) = \)
   (a) \( e^{2t}\sin(t) \)  
   (b) \( t\cos(3t) \)  
   (c) \( t^{5/2} \)  
   (d) \( f(t) = \begin{cases} t, & \text{if } 0 \leq t < 1 \\ t^2, & \text{if } t > 1 \end{cases} \)

4. Find the Laplace transform \( F(s) \) of the square wave \( f(t) \) with period 2, where
   \[ f(t) = \begin{cases} 1, & \text{if } 0 \leq t < 1 \\ -1, & \text{if } 1 < t < 2 \end{cases} \]

5. Find the inverse transforms \( f(t) \) of \( F(s) = \)
   (a) \( (3s+8)/(s^2 - 8s + 25) \)  
   (b) \( e^{-3s}/s^4 \)  
   (c) \( (s+3)/(s-1)^2(s^2+4) \)

6. Solve using Laplace transforms
   (a) \( y'' - 3y' + 2y = 4e^{2t}; \ y(0) = 0, \ y'(0) = 1 \)
   (b) \( y'' + 25y = 105(t-2); \ y(0) = y'(0) = 0 \)
   (c) \( y'' - 4y' = \begin{cases} 3, & \text{if } 0 < t < 2 \\ 0, & \text{if } 2 \leq t \end{cases} \); with initial values \( y(0) = 1, \ y'(0) = 0 \).

7. A mass of 4 grams on a spring with constant \( k=100 \) is released from rest at time \( t=0 \) 2 cm above equilibrium. Then at time \( t=3 \), the mass is given an upward impulse of power 120. Write the differential equation for the position \( x(t) \) of the mass at time \( t \) and use Laplace transforms to solve for \( x(t) \).

8. A rocket is launched with acceleration \( 68 - t^2 \) for time \( 0 \leq t \leq 10 \) and acceleration \( -32 \) for \( t \geq 10 \). Write the differential equation for the position \( x(t) \) of the rocket at time \( t \) and use Laplace transforms to solve for \( x(t) \).