Sample Problems for Exam Three

1. State and prove the formulas for
(a) $\mathcal{L}\{e^{at}f(t)\}$ (b) $\mathcal{L}\{f'(t)\}$
(c) $\mathcal{L}\{tf(t)\}$ (d) $\mathcal{L}\{u(t-a)f(t-a)\}$

2. Define the Gamma function $\Gamma(p+1)$ and show that $\mathcal{L}\{t^p\} = \Gamma(p+1)/s^{p+1}$.

3. Find the Laplace transform $F(s)$ of $f(t) =$
(a) $e^{2t} sint$ (b) $t\ cos\ 3t$ (c) $t^{5/2}$
(c) $\begin{cases} t, & \text{if } 0 \le t < 1 \\ t^2, & \text{if } t > 1 \end{cases}$.

4. Find the Laplace transform $F(s)$ of the square wave $f(t)$ with period 2, where

$$f(t) = \begin{cases} 1, & \text{if } 0 \le t < 1 \\ -1, & \text{if } 1 < t < 2 \end{cases}.$$

5. Find the inverse transforms f(t) of F(s) =
(a) $(3s+8)/(s^2-8s+25)$ (b) e^{-3s}/s^4 (c) $(s+3)/(s-1)^2(s^2+4)$

6. Solve using Laplace transforms
(a) $y'' - 3y' + 2y = 4e^{2t}$; $y(0) = 0$, $y'(0) = 1$
(b) $y'' + 25y = 10\delta(t-2)$; $y(0) = y'(0) = 0$
(c) $y'' - 4y' = \begin{cases} 3, & \text{if } 0 < t < 2 \\ 0 & \text{if } 2 \le t \end{cases}$; with initial values $y(0) = 1$, $y'(0) = 0$.

7. A mass of 4 grams on a spring with constant k=100 is released from rest at time t=0 2 cm above equilibrium. Then at time t=3, the mass is given an upward impulse of power 120. Write the differential equation for the position x(t) of the mass at time t and use Laplace transforms to solve for x(t).

8. A rocket is launched with acceleration $68 - t^2$ for time $0 \le t \le 10$ and acceleration -32 for $t \ge 10$. Write the differential equation for the position x(t) of the rocket at time t and use Laplace transforms to solve for x(t).