MAC 2311 Quiz 10 Lectures 23 - 24

Name: Jack Chou

1. (5 pts) Find the inflection points of the following function in the interval $[0, \pi]$

$$f(x) = \cos^2(x) - 2\sin(x)$$

$f'(x) = -2\cos(x)\sin(x) - 2\cos(x) = -2\cos(x)[\sin(x)+1]$

$f''(x) = 2\sin(x)[\sin(x)+1] - 2\cos(x)[\cos(x)]$

$= 2\sin^2(x) + 2\sin(x) - 2\cos^2(x)$

$f''(x) = 0 = 2\sin^2(x) + 2\sin(x) - 2(1-\sin^2(x))$

$\sin^2(x) + \sin(x) + \sin^2(x) - 1 = 2\sin^2(x) + \sin(x) - 1$

$= (2\sin(x)-1)(\sin(x)+1) = 0$

$\sin(x) = \frac{1}{2}$ or $\sin(x) = -1$, $x = \frac{\pi}{6}, \frac{5\pi}{6}$

2. (5 pts) Evaluate the following limit.

$$\lim_{x \to 1} \frac{x^a - 1}{x^2 - 1}$$

$\lim\limits_{x \to 1} x^a - 1 = 1^a - 1 = 0$

$\lim\limits_{x \to 1} x^2 - 1 = 1^2 - 1 = 0$

$\lim\limits_{x \to 1} \frac{x^a - 1}{x^2 - 1} = \lim\limits_{x \to 1} \frac{ax^{a-1}}{2x} = \frac{a(1)^{a-1}}{2\cdot(1)} = \frac{a}{2}$