MAC 2311   Quiz 1 Lectures 1 - 2

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1. Find all solutions to the following equation on the interval $0 \le x < 2\pi$

$$2\sin^2(x) - \sqrt{2}\sin(x) = 0$$

Let $a = \sin(x)$, then $2a^2 - \sqrt{2}a = 0$

$\Rightarrow a = \dfrac{\sqrt{2} \pm \sqrt{2-0}}{4}$ $\Rightarrow a = \dfrac{\sqrt{2}}{2}$ or 0 $\Rightarrow \sin(x) = \dfrac{\sqrt{2}}{2}$ or 0

If $\sin(x) = \dfrac{\sqrt{2}}{2}$, then $x = \dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$

If $\sin(x) = 0$, then $x = 0$ or π

$$\boxed{x = 0, \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \pi}$$

2. Find the inverse $f^{-1}(x)$ of $f(x)$ restricted to the domain of $x \ge 0$.

$$f(x) = \dfrac{3}{\sqrt{4x^2 + 9}}$$

$y = \dfrac{3}{\sqrt{4x^2+9}}$ $\Rightarrow$ $y^2 = \dfrac{9}{4x^2+9}$ $\Rightarrow$ $4x^2+9 = \dfrac{9}{y^2}$

$\Rightarrow 4x^2 = \dfrac{9}{y^2} - 9 = \dfrac{9 - 9y^2}{y^2} = \dfrac{9(1-y^2)}{y^2}$

$\Rightarrow x^2 = \dfrac{9(1-y^2)}{4y^2}$ $\Rightarrow$ $x = \dfrac{3\sqrt{1-y^2}}{2y}$

Therefore $\boxed{f^{-1}(x) = \dfrac{3\sqrt{1-x^2}}{2x}}$