MAC 2311 Quiz 1 L1-L2

Name:
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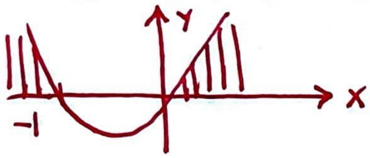
1. Solve the following inequality and describe the set determined by the condition.

$$|2+\frac{1}{x}|>1$$

i) $2+\frac{1}{x}>1$, multiplying both sides by x^2 (to ensure the inequality's direction remains unchanged).

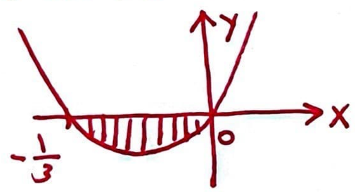
$\Rightarrow 2x^2+x>x^2 \Rightarrow x^2+x>0$



$\Rightarrow x<-1$ or $x>0$

ii) $2+\frac{1}{x}<-1$ ($\because -3<-1$ and $|-3|=3>1$)

$\Rightarrow 2x^2+x<-x^2 \Rightarrow 3x^2+x<0$ where $3x^2+x=x(3x+1)$



$-\frac{1}{3}<x<0$

$\therefore$ Hence, the answer is

$(-\infty,-1)\cup(-\frac{1}{3},0)\cup(0,\infty)$

2. When $f(x)=(x-1)^{\frac{1}{3}}$ and $g(x)=8x^3+1$, compute $(f\circ g)(x)$ and describe its domain.

1) Find $y=(f\circ g)(x)=??$

$$y=(f\circ g)(x)=f(g(x))=(g(x)-1)^{\frac{1}{3}}=((8x^3+1)-1)^{\frac{1}{3}}=(8x^3+1-1)^{\frac{1}{3}}$$

$$=(8x^3)^{\frac{1}{3}}=(8)^{\frac{1}{3}}\cdot(x^3)^{\frac{1}{3}}=(2^3)^{\frac{1}{3}}\cdot(x^3)^{\frac{1}{3}}=2^1\cdot x^1=2x.$$

2) The answer is $\mathbb{R}$. (or you can say $(-\infty,\infty)$.)

① when $x-1<0$, f can be defined. ($\because$ when $x=0$, $x-1=-1<0$, and $(-1)^{\frac{1}{3}}=-1$. well-defined!)

② If $x-1=0$, we can still define f as well.
($\because 0^{\frac{1}{3}}=0$)

But, if we consider a ftn $y=(x-1)^{-\frac{1}{3}}$, we need to be careful.
($\because$ when $x-1=0$, $(0)^{-\frac{1}{3}}=\frac{1}{0^{\frac{1}{3}}}=\frac{1}{0}$ (we can't define this.))

③ Thus, f can be defined on $(-\infty,\infty)$ and so is g. ($\because g$ is a polynomial function)
Therefore, we can define $(f\circ g)(x)$ on $(-\infty,\infty)$.