Instructor: Minseo Cho

1. Find a x-value where the given function has a horizontal tangent line.

$$y = x^3 - 9x^2 + 27x - 27.$$

Proof. Let $y = f(x) = x^3 - 9x^2 + 27x - 27$. We can find a horizontal tangent line when $f'(x) = 0$. Let's do it step by step!
we know that $\frac{df(x)}{dx} = f'(x) = (x^3 - 9x^2 + 27x - 27)' = (x^3)' + (-9x^2)' + (27x)' + (-27)'$.
And Here we have $\cdots$

- $(x^3)' = \frac{d}{dx}x^3 = 3 \cdot x^2$,
- $(-9x^2)' = \frac{d}{dx}(-9x^2) = -9 \cdot \frac{d}{x}x^2 = -9 \cdot (2x) = -18x$,
- $(27x)' = \frac{d}{dx}(27x) = 27 \cdot \frac{d}{dx}x = 27$,
- $(-27)' = \frac{d}{dx}(-27) = 0$.

Thus, we know that $\cdots$

$$\begin{aligned} f'(x) = \frac{df(x)}{dx} &= f'(x) = (x^3 - 9x^2 + 27x - 27)' \\ &= (x^3)' + (-9x^2)' + (27x)' + (-27)' \\ &= 3 \cdot x^2 + (-18x) + 27 + 0 \\ &= 3x^2 - 18x + 27 \\ &= 3 \cdot (x^2 - 6x + 9) = 3 \cdot (x-3)^2. \end{aligned}$$

Therefore, since $f'(x) = 3 \cdot (x-3)^2$, we can find that $y = f(x)$ has a horizontal tangent line at when $x = 3$. □

2. Find $f'(x)$ for the function $f(x) = (x-1)e^x$.

Proof. Say $f(x) = (x-1)$ and $g(x) = e^x$. First of all, note that $\cdots$

- $f'(x) = \frac{df(x)}{dx} = \frac{d}{dx}(x-1) = \frac{d}{dx}(x) - \frac{d}{dx}(1) = 1 - 0 = 1$,
- $g'(x) = \frac{dg(x)}{dx} = \frac{d}{dx}(e^x) = \frac{d}{dx}(e^x) = e^x$.

Now, applying the product rule,

$$\begin{aligned} (f(x)g(x))' = \frac{d}{dx}(f(x)g(x)) &= f'(x)g(x) + f(x)g'(x) \\ &= 1 \cdot e^x + (x-1) \cdot e^x \\ &= e^x + (x-1)e^x \\ &= \{1 + (x-1)\} \cdot e^x \\ &= x \cdot e^x. \end{aligned}$$

□