Instructor: Minseo Cho

1. Find a derivative of $f(x) = cos(x) + 2$ when $x = \frac{\pi}{3}$.

   Proof. We know that $\cdots$

   - $\frac{d}{dx}(cos(x)) = (cos(x))' = -sin(x)$,
   - $\frac{d}{dx}2 = (2)' = 0$ since 2 is a constant function.

   $\therefore$ Thus, since $\frac{d}{dx}(cos(x)+2) = -sin(x)$, $f'(\frac{\pi}{3}) = -sin(\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$. □

2. Suppose that a function $y = f(x)$ satisfies the following conditions:

   (i) $f(3) = 7$ and $f'(3) = 5$,

   (ii) $f(x) = f(-x+1) + 3$ for all x.

   By using the chain rule and conditions, find the tangent line of $f(x)$ when x is -2.

   Proof. when we find a tangent line at $x = -2$, we can write it as $\cdots$

   $$y = f'(-2)(x - (-2)) + f(-2) = f'(2)(x+2) + f(-2).$$

   Thus, what we should find are **(1) f′(2)** and **(2) f(-2)**.

   (i) since we have $f(x) = f(-x+1) + 3$ for all x, it implies that $\cdots$

   $f(-2) = f(-(-2)+1)+3 = f(3)+3; \quad f(-2) = f(3)+3 = 10$ which means $f(-2) = 10$.

   (ii) Use the chain rule for a given condition $f(x) = f(-x+1) + 3$.

   $$\begin{aligned}\frac{df}{dx} = f'(x) &= \frac{d}{dx}(f(-x+1)+3) \\ &= \frac{d}{dx}f(-x+1) + \frac{d(3)}{dx} \\ &= f'(-x+1)\cdot\frac{d}{dx}(-x+1) + 0 \\ &= f'(-x+1)\cdot(-1) \\ &= -f'(-x+1).\end{aligned}$$

   From the above equation, we find $\cdots$

   $$5 = f'(3) = -f'(-3+1) = -f'(-2); \quad f'(-2) = -5.$$

   $\therefore$ So we have $\cdots$

   $$y = f'(2)(x+2) + f(-2) = -5(x+2) + 10 = -5x.$$

   □