MAC 2311 Quiz 10 Solution L23-L24

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1. Compute the following limit.

$$\lim_{x\to 0+}(1+tan(2x))^{cot(2x)}$$

Proof. Let $y = f(x) = (1+tan(2x))^{cot(2x)}$. Then,after taking $ln(x)$, we have $\cdots$

$$ln(f(x)) = ln(1+tan(2x))^{cot(2x)} = cot(2x)ln(1+tan(2x)) = \frac{ln(1+tan(2x))}{tan(2x)},$$

since $cot(2x) = \frac{1}{tan(2x)}$. Recall the L'Hôpital's rule.

Suppose that

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0} \quad \text{or} \quad \frac{\pm\infty}{\pm\infty}$$

and that the derivatives $f'(x)$ and $g'(x)$ exist near $x = a$, and $g'(x) \neq 0$ near $x = a$, except possibly at a. If the limit

$$\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

exists (or is $\pm\infty$), then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}.$$

In this case, since both $tan(2x)$ and $ln(1+tan(2x)$ are closer to 0 when x goes to 0, we can apply L'Hôpital's Rule in this case. Then, we can compute that

$$\begin{aligned}
\lim_{x\to 0} lnf(x) = \lim_{x\to 0}\frac{ln(1+tan(2x))}{tan(2x)} &= \lim_{x\to 0}\frac{((ln(1+tan(2x))))'}{(tan(2x))'} \\
&= \lim_{x\to 0}\frac{\frac{2sec^2(2x)}{1+tan(2x)}}{2sec^2(2x)} \\
&= \lim_{x\to 0}\frac{2sec^2(2x)}{1+tan(2x)}\cdot\frac{1}{2sec^2(2x)} \\
&= \lim_{x\to 0}\frac{1}{1+tan(2x)} \\
&= \frac{1}{1+0} \\
&= 1
\end{aligned}$$

Thus, this means $\lim_{x \to 0} ln(f(x)) = 1$ which means $\lim_{x \to 0} f(x) = e$.

What I want to say about the L'Hôpital's rule is **to be careful when we use that.** We can apply this rule **only if they satisfies this condition**.

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \quad \text{or} \quad \frac{\pm\infty}{\pm\infty}$$

□

2. Find the inflection points of $y = x^4 - 4x^2 + 4$.

Proof. We can find the inflection points by using the condition $f''(x) = 0$ and comparing its sign. First, say $f(x) = x^4 - 4x^2 + 4$ and we have $\cdots$

$$f''(x) = (x^4 - 4x^2 + 4)'' = (4x^3 - 8x)' = 12x^2 - 8.$$

Once we set $f''(x) = 0$, then we can get inflection points which are $\sqrt{\frac{2}{3}}$ and $-\sqrt{\frac{2}{3}}$. □