MAC 2311 Quiz 6 L15-L16

Name:
Instructor: Minseo Cho

1. Let C be a circle whose center is $(-1, 0)$ and radius is $\sqrt{2}$.

$$C : (x+1)^2 + y^2 = 2. \iff (x+1)^2 + y^2 - 2 = 0$$

Let ℓ_1 be a tangent line at $(-2, 1)$. Then, find a tangent line ℓ_2 to a given circle C that is perpendicular to the tangent line ℓ_1 ← (*)

1) Find a tangent line ℓ_1 at $(-2,1)$

(∵ $\frac{d}{dx}\{(x+1)^2 + y^2 - 2\} = ((x+1)^2)' + (y^2)' + (-2)'$

$= 2(x+1) + (y^2)' + (-2)'$

$= 2(x+1) + 2y \cdot y' + 0$

$= 2(x+1) + 2y \cdot y'$

$\therefore\ 0 = \frac{d}{dx}(0) = \frac{d}{dx}\{(x+1)^2 + y^2 - 2\} = 2(x+1) + 2y \cdot y'$

$\Rightarrow 2y \cdot y' = -2(x+1) \Rightarrow y' = \frac{-2(x+1)}{2y} = \frac{-(x+1)}{y}$

Thus, a tangent slop ℓ_1 at $(-2,1)$ is $\cdots \frac{-(-2+1)}{1} = 1$

Hence, $\ell_1 : y = 1 \cdot (x+2) + 1 = x + 3.$)

2) If ℓ_2 is perpendicular to ℓ_1, when we say m_2 as a tangent slope of ℓ_2, we have $\cdots$

$$m_2 \cdot 1 = -1 \Rightarrow m_2 = -1$$

Now, we need to find some points P and Q on C whose derivatives have -1.

MAC 2311 Quiz 6 L15-L16

Name:
Instructor: Minseo Cho

1. Let C be a circle whose center is $(-1, 0)$ and radius is $\sqrt{2}$.

$$C : (x+1)^2 + y^2 = 2.$$

Let ℓ_1 be a tangent line at $(-2, 1)$. Then, find a tangent line ℓ_2 to a given circle C that is perpendicular to the tangent line ℓ_1

3) Since P and Q're on a circle C and have a tangent slope 1, they are satisfied the following condition.

(i) $(x+1)^2 + y^2 = 2$ (ii) $\frac{dy}{dx} = \frac{-(x+1)}{y} = -1$

$\Leftrightarrow x+1 = y.$

$\Rightarrow$ From (i), $(x+1)^2 + y^2 = 2$ $y = x+1$

$(x+1)^2 + (x+1)^2 = 2$

$2 \cdot (x+1)^2 = 2$; $(x+1)^2 = 1$

which implies $x = -2$ or 0.

When $x = -2$, $y = x+1 = -2+1 = -1$; $P = (-2, -1)$

when $x = 0$, $y = x+1 = 0+1 = 1$; $Q = (0, 1)$

$\therefore$ Hence, ℓ_2're ...

① $y = -1 \cdot (x+2) - 1 = -x - 3$

② $y = -1(x-0) + 1 = -x + 1$ ∎

MAC 2311 Quiz 6 L15-L16

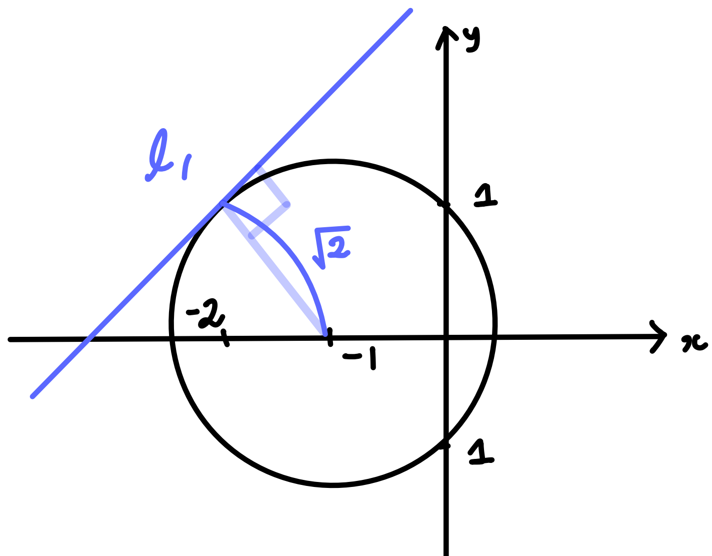
Name:
Instructor: Minseo Cho

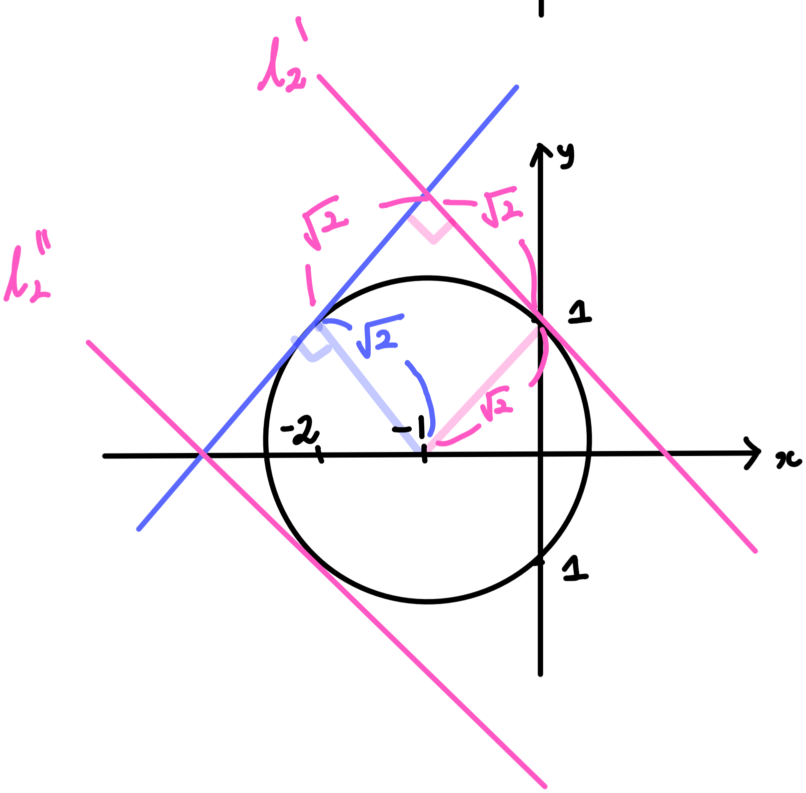
1. Let C be a circle whose center is $(-1, 0)$ and radius is $\sqrt{2}$.

$$C : (x+1)^2 + y^2 = 2.$$

Let ℓ_1 be a tangent line at $(-2, 1)$. Then, find a tangent line ℓ_2 to a given circle C that is perpendicular to the tangent line ℓ_1

* Or .. You can solve this by using geometry.





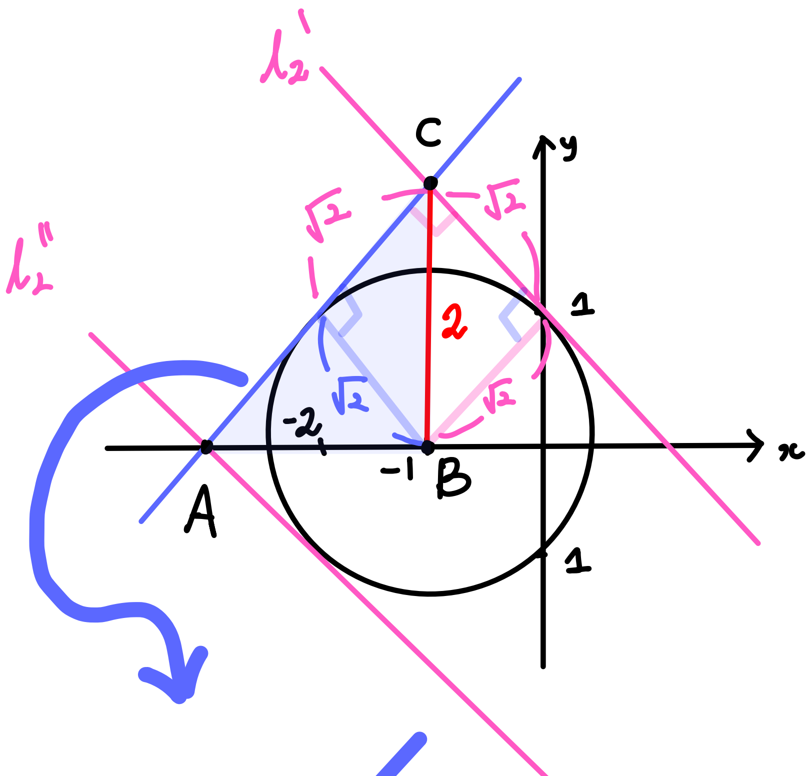
Name:

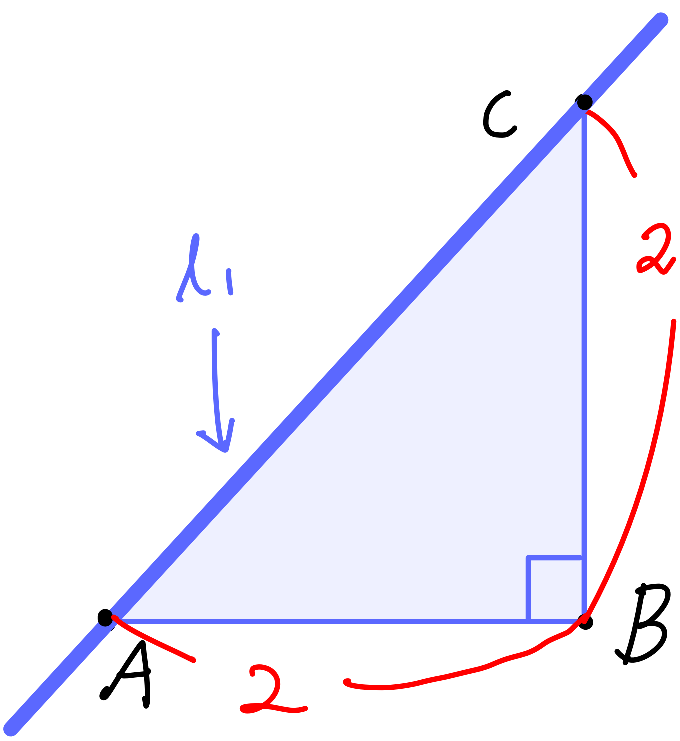
Instructor: Minseo Cho

1. Let C be a circle whose center is $(-1, 0)$ and radius is $\sqrt{2}$.

$$C : (x+1)^2 + y^2 = 2.$$

Let ℓ_1 be a tangent line at $(-2, 1)$. Then, find a tangent line ℓ_2 to a given circle C that is perpendicular to the tangent line ℓ_1





Since a slope of a tangent line l_1 is 1, it means $\frac{y\text{-value change}}{x\text{-value change}} = \frac{1}{1} = \frac{2}{2}$

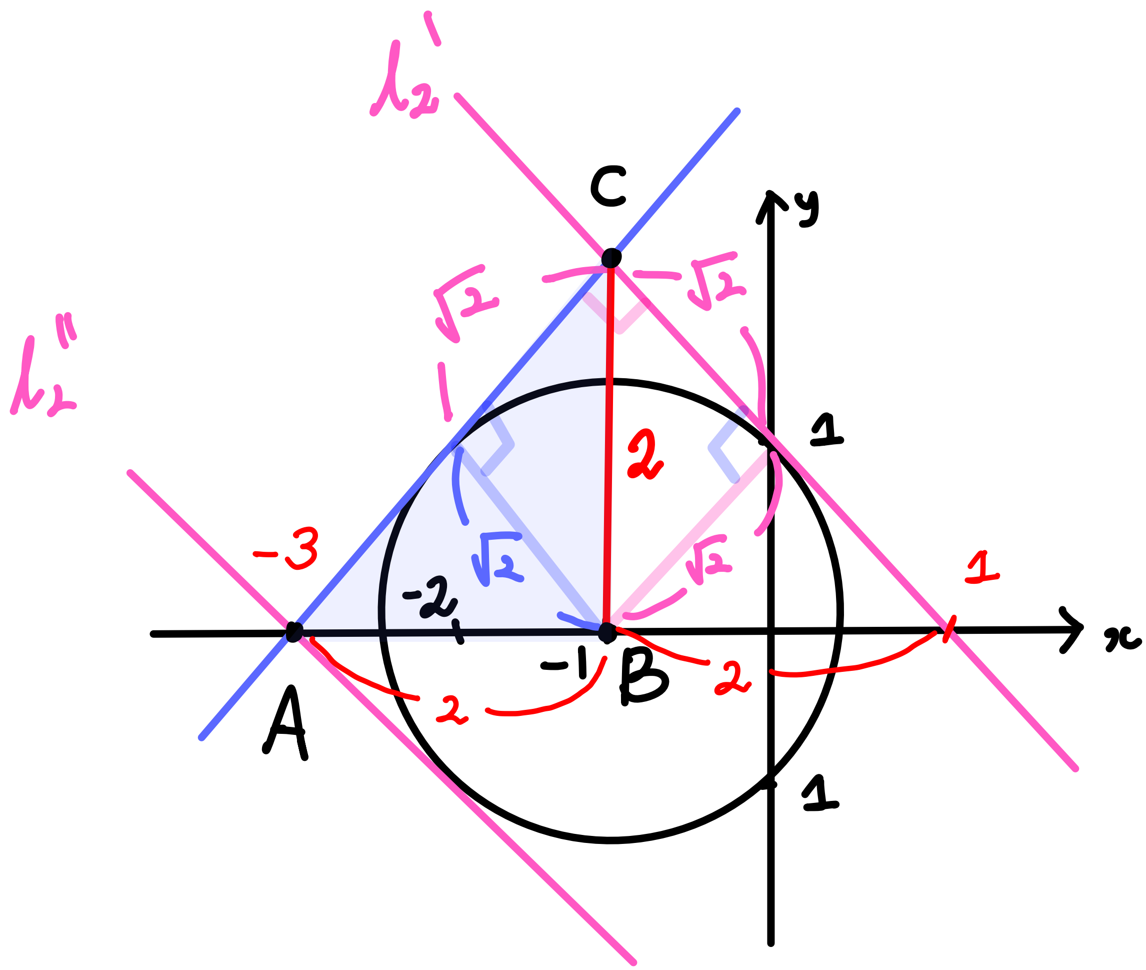
Thus, since $\overline{BC} = 2$, a length of $\overline{AB}$ is 2 as well.

Name:
Instructor: Minseo Cho

1. Let C be a circle whose center is $(-1, 0)$ and radius is $\sqrt{2}$.

$$C : (x+1)^2 + y^2 = 2.$$

Let ℓ_1 be a tangent line at $(-2, 1)$. Then, find a tangent line ℓ_2 to a given circle C that is perpendicular to the tangent line ℓ_1



From this figure, we can verify that l_2're ...

① $y = -(x+3) = -x-3$

② $y = -(x-1) = -x+1.$

2. Let $y = f(x) = cos(2x)$. Then compute its 2025th derivative of $f(x)$.
   (In other words, find what $\frac{d^{2025}f}{dx^{2025}}$ is.)

$$f'(x) = -2\cdot \sin(2x) = -2^{1}\cdot \sin 2x$$

$$f''(x) = -2\cdot (\sin(2x))' = -2\cdot 2\cdot \cos(2x) = -2^{2}\cdot \cos 2x$$

$$f'''(x) = -2\cdot 2\cdot (\cos(2x))' = -2\cdot 2\cdot (-2\cdot \sin(2x)) = 2^{3}\cdot \sin(2x)$$

$$f''''(x) = -2\cdot 2\cdot (-2)\cdot (\sin(2x))' = -2\cdot 2\cdot (-2)\cdot 2\cdot \cos(2x) = -2^{4}\cdot \cos 2x$$

And remind that $\frac{d^{4k+1}}{dx^{4k+1}}\cos(x) = -\sin x$

(Review Lecture 14 homework :)!)

Since $2025 = 506 \times 4 + 1$,

$$\frac{d^{2025}}{dx^{2025}}(\cos(2x)) = -2^{2025}\cdot \sin(2x).$$