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1. Let $y = f(x) = |x|$ be a function on a closed interval $[-1, 1]$

   (a) Can we apply the mean value theorem for this function on the closed interval [0,1]? If so, provide your reasoning.

   Proof. Remind the mean value theorem.

   > Let $f : [a, b] \to \mathbb{R}$ be a continuous function on the closed interval $[a, b]$ and differentiable on the open interval (a, b), where $a < b$. Then there exists some c in (a, b) such that $\cdots$
   >
   > $$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

   This means that if the given function is continuous on $[0, 1]$ and differentiable on $(0, 1)$, we can apply the above theorem.
   In general, we cannot use the mean value theorem for a function $y = f(x) = |x|$ since this function is not always differentiable on every open interval. For example, if we see an open interval $I = (-1, 1)$, since it cannot be differentiable at $x = 0$, we cannot apply theorem. **But, since $y = f(x)$ is differentiable on an open interval** $(0, 1)$, so we can.

   $\therefore$ Therefore, in this case, we can use the mean value theorem. □

   (b) From (a), how many points c satisfy the mean value theorem?

   A. One

   B. Infinitely many

   C. None

   Proof. If we apply the mean value theorem, it implies that there exists some $c \in (0, 1)$ such that $f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{1-0}{1-0} = 1$. Recall that $y = f(x) = x$ when x is nonnegative. Thus, it means that $f'(x) = 1$ for all $x \in [0, \infty)$. Therefore, we can find that $f'(x) = 1$ for all $x \in [0, 1]$. Since the number of elements in $[0, 1]$ is infinity, so the answer is **B**. □