

Drawing the graph of a function $y = f(x) = x^2 e^x$.

1) Domain: $(-\infty, \infty)$

2) Differentiation.

$$y' = f'(x) = (x^2 e^x)' = 2x \cdot e^x + x^2 e^x = (2x + x^2) e^x = x \cdot (x+2) \cdot e^x.$$

(∴ Product rule)

Since $e^x > 0$, we can find that ...

i) When $x < -2$ or $x > 0$, $f'(x) > 0$

⇒ Increasing.

ii) When $-2 < x < 0$, $f'(x) < 0$

⇒ Decreasing.

3) Points: $f(0) = 0 \Rightarrow (0, 0)$

$$f(-2) = 4 \cdot e^{-2}$$

4) Asymptote: $\lim_{x \rightarrow -\infty} x^2 e^x = 0$ Since $\lim_{x \rightarrow -\infty} e^x = 0$.

