1. Let $y = f(x) = xe^x$, and let ℓ be the tangent line to $f(x)$ at a critical point. Consider the following points in the xy plane:

   - $A(a, f(a))$, where a is a critical point of $f(x)$,
   - $B(0, b)$, where b is the y-intercept of the tangent line ℓ,
   - $O(0, 0)$, the origin.

   Find the area of the triangle $\triangle OAB$.

<sol> First, we need to find a critical point of $f(x)$.

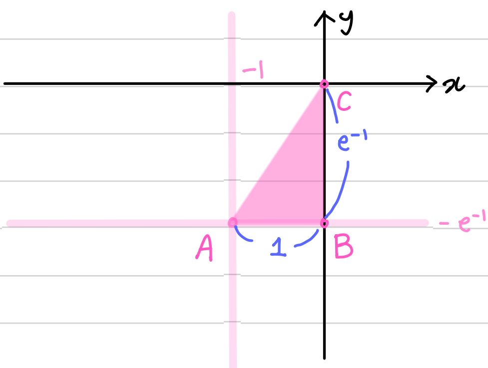
- $f'(x) = (x \cdot e^x)' = 1 \cdot e^x + x \cdot e^x = (1+x)e^x$; when $x=-1$, $f'(-1)=0$

  (∵ Product rule)

Since $f'(-1)=0$ where $x=-1$ is a critical tangent line,

a tangent line l_1 is a horizontal line passing through $(-1, f(-1)) = (-1, -e^{-1})$

Thus, the y-intersect of l_1 is $(0, -e^{-1})$



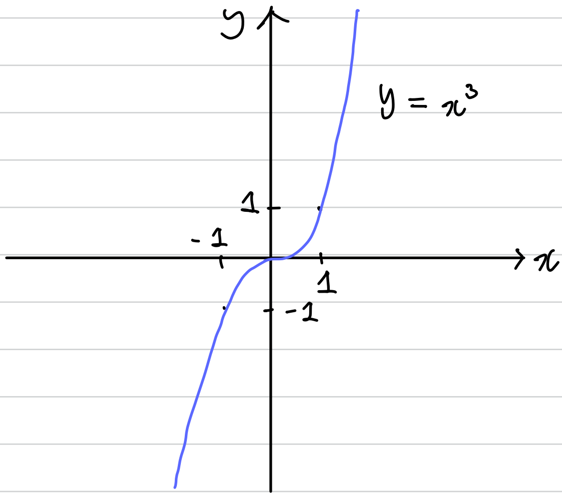
So, from this graph, we can check the area of $\triangle ABC$ is ...

$\frac{1}{2} \cdot 1 \cdot e^{-1} = \frac{1}{2} \cdot 1 \cdot \frac{1}{e} = \frac{1}{2e}$ $\blacksquare$

2. Let $y = f(x)$ be a function that has a critical value at $x = c$. Does the function f necessarily have an extreme value at $x = c$? Justify your answer briefly. (If false, provide a counterexample.)

The answer is "False".

Consider $y = f(x) = x^3$. Since $y' = f'(x) = 3x^2$, it has a critical value at $x=0$.
But, since $f'(x) \geq 0$ for all $x \in (-\infty, \infty)$, it keep increasing.
$\Rightarrow$ It does not mean that $f(0)$ is the extreme value on $(-\infty, \infty)$.



Drawing the graph of the above ftns

1) $y = x \cdot e^x$

(1) Domain. $(-\infty, \infty)$

(2) Differentiation: $y' = 1 \cdot e^x + x \cdot e^x = (1+x)e^x$

Since $e^x > 0$, We can see that …

When $x < -1$, $(1+x)e^x < 0 \Rightarrow$ Decreasing

When $x > -1$, $(1+x)e^x > 0 \Rightarrow$ Increasing

(3) Points: When $x=0$, $f(0)=0 \Rightarrow (0,0)$
when $x=-1$, $f(-1) = -1/e \Rightarrow (-1, -1/e)$

(4) Asymptote: Since $\lim_{x \to -\infty} e^x = 0$, $\lim_{x \to -\infty} xe^x = 0$