# Lecture 17: Series (II)

## Telescoping Series

A telescoping series is a special type of series for which many terms cancel in the $n^{th}$ partial sums.

One way to determine whether a telescoping series converges or diverges, we write out the $n^{th}$ partial sums of the series. And if the limit of the partial sum is finite, then it converges, and we can find out the exact sum of the series.

1. Find Partial Fraction (if $a_n$ is rational expression)
2. Compute $S_n$
3. Find $\lim_{n\to\infty} S_n$

Determine if the following series converge.
If converge, find the sum:

**<u>ex.</u>** 1. $$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

**ex.** 2. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$.

Students in this class are expected to know the telescoping series even it is not in the text.

**Theorem:** If $\sum a_n$ and $\sum b_n$ are <u>both convergent</u> series,then so are $\sum ca_n$, ($c$ any constant), $\sum(a_n \pm b_n)$, and

$$\sum ca_n = c \sum a_n,$$

$$\sum a_n \pm b_n = \sum a_n \pm \sum b_n$$

**ex.** 3. $\displaystyle\sum_{n=1}^{\infty} \left( \frac{3}{n(n+3)} + \frac{1}{2^n} \right)$.

**NYTI:** 1. show that the series $\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$ converges and find the sum it converges to.

2. Use partial fractions to find the sum of the series $\sum_{n=2}^{\infty} \frac{1}{n^3-n}$. $(\frac{1}{4})$

3. Find the sum of the series $\sum_{n=2}^{\infty} \ln\left(1-\frac{1}{n^2}\right)$.

4. Find the values of $x$ for which the following series converges. Find the sum of the series for these values of $x$.

$$\sum_{n=0}^{\infty} \frac{(x+3)^n}{2^n}$$

$(-5 < x < -1$,converges to $\frac{2}{-1-x})$

5. Suppose both $\sum_{n=1}^{\infty} a_n, \sum_{n=1}^{\infty} b_n$ diverge. Then $\sum_{n=1}^{\infty}(a_n - b_n)$ diverges. True or False?

6. Sum of a divergent and a convergent series must diverge. True or False?

7. Determine the convergence of the series and find the sum if it converges.

$$\sum_{n=2}^{\infty} \frac{-1+(-3)^n}{4^n}$$