

Lecture 31: Parametric Curves (11.1, 11.2)

Consider a parametric curve defined by the parametric equations $x(t)$ and $y(t)$ on an interval.

Parametric Formula for dy/dx :

If all three derivatives, dy/dt , dx/dt , and dy/dx exist and $dx/dt \neq 0$,

then

$$\boxed{\frac{dy}{dx} = \frac{dy/dt}{dx/dt}}$$

The second derivative is given by

$$\boxed{\frac{d^2y}{dx^2} = \frac{d(dy/dx)}{dx} = \frac{d(dy/dx)/dt}{dx/dt}}$$

ex. Consider the parametric curve

$$x = \sqrt{t}, \quad y = (t^2 - 4) / 4, \quad t > 0$$

Find the slope and concavity at the point $(2, 3)$.

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ex. Find an equation of the tangent line to the parametric curve at the point corresponding to the given value of the parameter.

$$x = 2t^2 + 1, \quad y = \frac{1}{3}t^3 - t, \quad t = 3 \quad (y = \frac{2}{3}x - \frac{20}{3})$$

ex. Find the points on the curve where the tangent line is horizontal or vertical.

$$x = 2 \cos t, \quad y = \sin 2t$$

(HTL at $(\sqrt{2}, 1), (-\sqrt{2}, -1), (-\sqrt{2}, 1), (\sqrt{2}, -1)$. VTL at $(2, 0), (-2, 0)$)

Arc Length

Suppose $x(t)$ and $y(t)$ are parametric equations that are continuous on $[\alpha, \beta]$ and its parametric curve C is traversed exactly once as t increases from α to β .

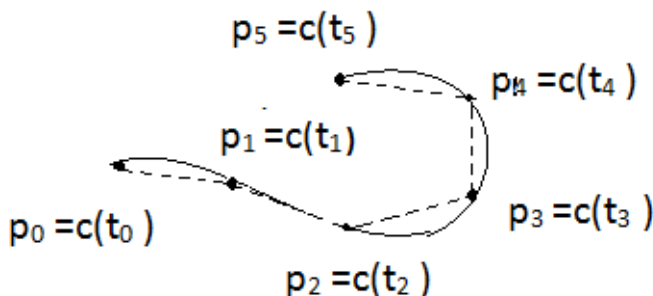
Recall that the distance between two points is given by

$$\begin{aligned} D &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(\Delta x)^2 + (\Delta y)^2} \end{aligned}$$

The Arc Length Formula We declare the arclength of a parametric curve from $t = \alpha$ to $t = \beta$ to be:

$$L = \lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{\left(\frac{\Delta x_i}{\Delta t}\right)^2 + \left(\frac{\Delta y_i}{\Delta t}\right)^2} \Delta t$$

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$



ex. Find the length of the curve

$$x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \leq t \leq 1 \quad (2(2\sqrt{2} - 1))$$

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1. A curve is defined by $x = t^2, y = t^3 - 3t$,

i. Find the points on the curve where the tangent line is horizontal or vertical

ii. Determine where the curve is concave up or down.

2. Find the t , if any, for which the curve traced by the parametric curve

$$x = \frac{1}{2}t^2 - 2t, \quad y = \frac{1}{3}t^3 - 4t$$

has HTL and VTL.

3. Given the parametric equation

$$x = \cos^2 t + \cos t, \quad y = \sin t \cos t + \sin t,$$

determine the t values where the curve has HTL and VTL on the interval of $0 \leq t \leq 2\pi$.

4. Calculate the arc length of a circle of radius R .