Abstract Algebra 1

MAS 4301

Section 8385

Exam 1

May 26, 2017

Please answer the following questions on the scratch paper provided. Be sure to put your name at the top of each page of your work. You may not use any books, notes, or calculators on this exam. You are required to show sufficient work for each question on the exam in order to receive full credit. You may use results we proved in class as long as you make it clear that you are doing so and you state them clearly.

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1. (9 points) Define what is a *group*. Give an example of a group, and an example of a set with a binary operation that is not a group.

2. (8 points) Let G be a group and let $g \in G$. Define what is the *order* of the element g. If $g^n = e$, does this mean that $|g| = n$? Explain why or why not.

3. (7 points) Let G be a group and $\mathbf{Z}(G) = \{z \in G \mid zg = gz \;\; \forall g \in G\}$. Prove that $\mathbf{Z}(G)$ is a subgroup of G.

4. (8 points) Prove that a group G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1} \;\; \forall a, b \in G$.

5. (8 points) Let $S = \mathbb{R}^3 \setminus \{\langle 0,0,0\rangle\}$. For $\mathbf{x}, \mathbf{y} \in S$, define $\mathbf{x} \sim \mathbf{y}$ if $\exists \lambda \in \mathbb{R}^*$ such that $\mathbf{y} = \lambda \mathbf{x}$. Show that $\sim$ is an equivalence relation on S, and describe geometrically the equivalence class of $\mathbf{x}$.

6. (10 points) Let $G = \mathbb{Z}_{36}$. List all of the generators of G. How many subgroups does G have? Give a generator for each subgroup of G. How many elements of order 9 does G have?