UF UNIVERSITY of FLORIDA

Department of Mathematics

MAC 2313
Exam 1A
Summer C 2015

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) Section number

C. Under "special codes" code in the test ID numbers 1, 1.

● 2 3 4 5 6 7 8 9 0
● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. **1)** This test consists of 15 multiple choice questions, worth 5 points each, plus two sheets (three pages) of free response questions worth 35 points. The test is counted out of 110 points, and there are 10 bonus points available.

2) The time allowed is 90 minutes.

3) You may write on the test.

4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tearoff sheets to your discussion leader or exam proctor. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted in Sakai (e-Learning) within one day after the exam. Your instructor will return your tearoff sheet with your exam score in class. Your score will also be posted in e-Learning within one week of the exam.

NOTE: Be sure to bubble the answers to questions 1–15 on your scantron.

Questions 1 – 15 are worth 5 points each.

1. Find the decomposition $\mathbf{u} = \mathbf{u}_{\parallel} + \mathbf{u}_{\perp}$ for the vector $\mathbf{u} = \langle 4, -1, 0\rangle$ with respect to the vector $\mathbf{v} = \langle 0, 1, 1\rangle$.

a. $\mathbf{u}_{\parallel} = \left\langle 0, -\frac{1}{2}, \frac{1}{2}\right\rangle$ $\qquad$ $\mathbf{u}_{\perp} = \left\langle 4, -\frac{1}{2}, -\frac{1}{2}\right\rangle$

b. $\mathbf{u}_{\parallel} = \left\langle 0, \frac{1}{2}, \frac{1}{2}\right\rangle$ $\qquad$ $\mathbf{u}_{\perp} = \left\langle 4, -\frac{3}{2}, -\frac{1}{2}\right\rangle$

c. $\mathbf{u}_{\parallel} = \left\langle 0, -\frac{1}{2}, -\frac{1}{2}\right\rangle$ $\qquad$ $\mathbf{u}_{\perp} = \left\langle 4, -\frac{1}{2}, \frac{1}{2}\right\rangle$

d. $\mathbf{u}_{\parallel} = \left\langle 0, -\frac{3}{2}, -\frac{3}{2}\right\rangle$ $\qquad$ $\mathbf{u}_{\perp} = \left\langle 4, \frac{1}{2}, \frac{3}{2}\right\rangle$

e. $\mathbf{u}_{\parallel} = \left\langle 0, -\frac{1}{2}, -\frac{1}{2}\right\rangle$ $\qquad$ $\mathbf{u}_{\perp} = \left\langle 4, \frac{1}{2}, -\frac{1}{2}\right\rangle$

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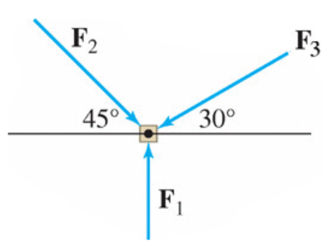
2. Three planar forces act on a particle as depicted below.



The magnitude of the forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are 3, $3\sqrt{2}$, and $6\sqrt{3}$ newtons respectively. What is the net force acting on the particle?

a. $\langle 12, 6 - 3\sqrt{3}\rangle$ $\qquad$ b. $\langle -6, 3 - 3\sqrt{3}\rangle$ $\qquad$ c. $\langle 6, -3\sqrt{3}\rangle$

d. $\langle -6, -3\sqrt{3}\rangle$ $\qquad$ e. $\langle 12, -3\sqrt{3}\rangle$

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3. If $\mathbf{u}$ and $\mathbf{v}$ are vectors and θ is the angle between $\mathbf{u}$ and $\mathbf{v}$, which of the following statements are true?

P. $\theta = \cos^{-1}\left(\dfrac{|\mathbf{u}\cdot\mathbf{v}|}{\|\mathbf{u}\|\|\mathbf{v}\|}\right)$

Q. If $\mathbf{u}\cdot\mathbf{v} < 0$, then θ is an obtuse angle.

R. $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{u} = 0$

S. If $\mathbf{u} = \lambda\mathbf{v}$ for some non zero scalar λ, then $\theta = 0$.

a. Q only

b. Q and R only

c. R only

d. P and Q only

e. Q, R, and S only

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4. The equation of the plane containing the point $(2, 0, -1)$ and the vectors $\langle 1, 2, 3\rangle$ and $\langle 4, -1, -2\rangle$ is given by

a. $7x - 14y + 9z = 5$

b. $7x + 14y + 9z = 5$

c. $-x + 14y - 9z = 7$

d. $-x - 14y + 9z = 7$

e. $x + 14y - 9z = 11$

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5. The normal vector to the plane $3x - 8y + 5z = 0$ is orthogonal to which of the following vectors?

a. $\langle -1, -1, 1\rangle$

b. $\langle 1, 1, 1\rangle$

c. $\langle 6, -16, 10\rangle$

d. $\langle 4, 1, -4\rangle$

e. None of the above

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6. If $\lambda \in \mathbb{R}$ is a scalar, which of the following statements are true?

P. $\|(\mathbf{i} \times \mathbf{j}) \times \mathbf{j}\| = 1$

Q. $(-\mathbf{j}) \times \mathbf{k} = \mathbf{i}$

R. $\mathbf{k}$ is orthogonal to $\mathbf{j} \times \mathbf{i}$

S. $\|\lambda\mathbf{i} \times (\mathbf{j} + \mathbf{k})\| = \lambda$

a. P only

b. P and S only

c. P, Q, and S only

d. Q, R, and S only

e. Q and S only

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7. A force $\mathbf{F} = \langle -3, 1, -2 \rangle$ newtons is applied to a particle, moving it from the point $P = (1, 3, 0)$ to $Q = (-1, 1, 4)$, where the distances are given in meters. Find the work done by the force $\mathbf{F}$.

a. 0 Nm

b. 4 Nm

c. -4 Nm

d. 12 Nm

e. -12 Nm

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8. Let $\mathbf{r}(t) = \langle \cos^2(t), \sin^2(t) \rangle$. Find the unit tangent vector $\hat{\mathbf{T}}(t)$.

a. $\langle -2\sin(t)\cos(t), 2\sin(t)\cos(t) \rangle$

b. $\langle \cos(t), \sin(t) \rangle$

c. $\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

d. $\left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$

e. None of the above

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9. Find the location at $t = 4$ of a particle whose path satisfies

$$\mathbf{r}'(t) = \langle 2t^{-1/2}, 6, 8t \rangle$$

and $\mathbf{r}(1) = \langle 4, 9, 2 \rangle$.

a. $\langle 8, 27, 62 \rangle$

b. $\langle 8, 62, 25 \rangle$

c. $\langle 8, 27, 60 \rangle$

d. $\langle 27, 8, 62 \rangle$

e. None of the above

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10. Let $\mathbf{r}(t) = \langle 3\cos(t), 5\sin(t), 4\cos(t) \rangle$. Which of the following statements is true?

a. $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are parallel.

b. $\|\mathbf{r}(t)\|$ is not a constant.

c. $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are not orthogonal

d. $\|\mathbf{r}(t)\|$ is a constant and $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are orthogonal

e. None of the above

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11. Find a parametrization of the tangent line to $\mathbf{r}(t) = \langle 1 - t^2, 5t, 2t^3 \rangle$ at the point $t = 2$.

a. $\mathbf{L}(t) = \langle 3 - 4t, -10 - 5t, 16 - 12t \rangle$

b. $\mathbf{L}(t) = \langle -3 + 4t, 10 - 5t, 16 + 24t \rangle$

c. $\mathbf{L}(t) = \langle 3 + 4t, -5 - 2t, -16 - 24t \rangle$

d. $\mathbf{L}(t) = \langle 3 - 4t, -10 - 5t, -16 - 24t \rangle$

e. $\mathbf{L}(t) = \langle -3 - 4t, 10 + 5t, 16 + 24t \rangle$

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12. The position of a particle is described by the vector function $\mathbf{r}(t) = \langle e^{t-2}, 12, 4t^{-2} \rangle$, where the distances are measured in meters. Calculate the speed of the particle at $t = 2$ seconds.

a. 2 ms^{-1} b. $\sqrt{2} \text{ ms}^{-1}$ c. $2\sqrt{2} \text{ ms}^{-1}$

d. 4 ms^{-1} e. $\frac{\sqrt{2}}{2} \text{ ms}^{-1}$

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13. Find the length of the curve $\mathbf{r}(t) = \langle -3t, 3, 4t + 6 \rangle$ over the interval $0 \le t \le 9$.

a. 50 b. 45 c. 40

d. 35 e. None of the above

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14. Let $\mathcal{C}$ be the curve whose equation is given by $\mathbf{r}(t) = \left\langle \sqrt{2}t, e^t, e^{-t} \right\rangle$. Compute $\hat{\mathbf{B}}(0)$, the unit binormal vector to $\mathcal{C}$ at $t = 0$.

a. $\left\langle -\dfrac{\sqrt{2}}{2}, \dfrac{1}{2}, -\dfrac{1}{2} \right\rangle$ b. $\left\langle \dfrac{\sqrt{2}}{2}, \dfrac{1}{2}, -\dfrac{1}{2} \right\rangle$ c. $\left\langle \dfrac{-1}{2}, \dfrac{\sqrt{2}}{2}, \dfrac{1}{2} \right\rangle$

d. $\left\langle \dfrac{\sqrt{2}}{2}, -\dfrac{1}{2}, \dfrac{1}{2} \right\rangle$ e. $\left\langle \dfrac{\sqrt{2}}{2}, \dfrac{1}{2}, \dfrac{1}{2} \right\rangle$

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15. Compute the principal unit normal vector $\hat{\mathbf{N}}(t)$ for $\mathbf{r}(t) = \langle t, \cos(t), \sin(t) \rangle$.

a. $\langle 1, -\cos(t), \sin(t) \rangle$ b. $\langle -1, \sin(t), -\cos(t) \rangle$

c. $\langle 0, -\cos(t), \sin(t) \rangle$ d. $\langle 1, \cos(t), -\sin(t) \rangle$

e. $\langle 0, -\cos(t), -\sin(t) \rangle$

MAC 2313 Exam 1A, Part II Free Response

Name: ________________________________ UF ID #: __________________

Signature: ________________________________ Section #: ________________

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (a) (2 points) Find the vector equation $\mathbf{r}_1(t)$ of the line passing through the points $(0, 1, 1)$ and $(1, 1, 2)$.

(b) (2 points) Find the vector equation $\mathbf{r}_2(s)$ of the line passing through the point $(2, 0, 3)$ and parallel to the line passing through points $(1, 3, 2)$ and $(0, -1, -2)$.

(c) (4 points) Do the lines $\mathbf{r}_1(t)$ and $\mathbf{r}_2(s)$ intersect? If so, find the point of intersection.

2. (8 points) Find the equation of the line of intersection of the planes $x - y + z = 4$ and $x + y - 3z = 2$.

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3.

(a) (1 point) Identify the quadric surface represented by the equation

$$x^2 + \frac{y^2}{16} + z^2 = 1$$

(b) (2 points) Describe the trace obtained by intersecting the above surface with the plane $y = 5$.

4. (4 points) Define the vector function $\mathbf{r}(t)$ by

$$\mathbf{r}(t) = \begin{cases} \left\langle \dfrac{\sin(2t)}{t}, t^2, e^t \right\rangle & \text{when } t \neq 0 \\ \langle 1, 0, 1 \rangle & \text{when } t = 0 \end{cases}$$

Is $\mathbf{r}(t)$ continuous for all t? Justify your answer.

5. (6 points) Calculate the curvature of the curve $\mathbf{r}(t) = \langle t, \sin(2t) \rangle$ at $t = \dfrac{\pi}{2}$.

6. (6 points) Find the components of acceleration $\mathbf{a_N}$ and $\mathbf{a_T}$ for the position vector $\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$.